

## Chapter 9 Linear and Quadratic Inequalities

### Section 9.1 Linear Inequalities in Two Variables

#### Section 9.1 Page 472 Question 1

**a)**  $y < x + 3$

Try (7, 10).

Left Side	Right Side
-----------	------------

$y$	$x + 3$
$= 10$	$= 7 + 3$
	$= 10$

Left Side  $\not<$  Right Side

Try (6, 7).

Left Side	Right Side
-----------	------------

$y$	$x + 3$
$= 7$	$= 6 + 3$
	$= 9$

Left Side  $<$  Right Side

Try (-7, 10).

Left Side	Right Side
-----------	------------

$y$	$x + 3$
$= 10$	$= -7 + 3$
	$= -4$

Left Side  $\not<$  Right Side

Try (12, 9).

Left Side	Right Side
-----------	------------

$y$	$x + 3$
$= 9$	$= 12 + 3$
	$= 15$

Left Side  $<$  Right Side

The ordered pairs (6, 7) and (12, 9) are solutions to the inequality  $y < x + 3$ .

**b)**  $-x + y \leq -5$

Try (2, 3).

Left Side	Right Side
-----------	------------

$-x + y$	$-5$
$= -2 + 3$	
$= 1$	

Left Side  $\not\leq$  Right Side

Try (-6, -12).

Left Side	Right Side
-----------	------------

$-x + y$	$-5$
$= -(-6) + (-12)$	
$= -6$	

Left Side  $\leq$  Right Side

Try (4, -1).

Left Side	Right Side
-----------	------------

$-x + y$	$-5$
$= -4 + (-1)$	
$= -5$	

Left Side  $\leq$  Right Side

Try (8, -2).

Left Side	Right Side
-----------	------------

$-x + y$	$-5$
$= -8 + (-2)$	
$= -6$	

Left Side  $\leq$  Right Side

The ordered pairs (-6, -12), (4, -1) and (8, -2) are solutions to the inequality  $-x + y \leq -5$ .

**c)**  $3x - 2y > 12$

Try (6, 3).

Try (12, -4).

Left Side	Right Side
$3x - 2y$	12
$= 3(6) - 2(3)$	
$= 12$	

Left Side  $\not>$  Right Side

Try  $(-6, -3)$ .

Left Side	Right Side
$3x - 2y$	12
$= 3(-6) - 2(-3)$	
$= -12$	

Left Side  $\not>$  Right Side

Left Side	Right Side
$3x - 2y$	12
$= 3(12) - 2(-4)$	
$= 44$	

Left Side  $>$  Right Side

Try  $(5, 1)$ .

Left Side	Right Side
$3x - 2y$	12
$= 3(5) - 2(1)$	
$= 13$	

Left Side  $>$  Right Side

The ordered pairs  $(12, -4)$  and  $(5, 1)$  are solutions to the inequality  $3x - 2y > 12$ .

**d)**  $2x + y \geq 6$

Try  $(0, 0)$ .

Left Side	Right Side
$2x + y$	6
$= 2(0) + 0$	
$= 0$	

Left Side  $\not\geq$  Right Side

Try  $(-4, -2)$ .

Left Side	Right Side
$2x + y$	6
$= 2(-4) + (-2)$	
$= -10$	

Left Side  $\not\geq$  Right Side

Try  $(3, 1)$ .

Left Side	Right Side
$2x + y$	6
$= 2(3) + 1$	
$= 7$	

Left Side  $\geq$  Right Side

Try  $(6, -4)$ .

Left Side	Right Side
$2x + y$	6
$= 2(6) + (-4)$	
$= 8$	

Left Side  $\geq$  Right Side

The ordered pairs  $(3, 1)$  and  $(6, -4)$  are solutions to the inequality  $2x + y \geq 6$ .

## Section 9.1 Page 472 Question 2

**a)**  $y > -x + 1$

Try  $(1, 0)$ .

Left Side	Right Side
$y$	$-x + 3$
$= 0$	$= -1 + 3$
	$= 2$

Left Side  $\not>$  Right Side

Try  $(-2, 1)$ .

Left Side	Right Side
$y$	$-x + 3$
$= 1$	$= -(-2) + 3$
	$= 5$

Left Side  $\not>$  Right Side

Try (4, 7).

Left Side	Right Side
$y$	$-x + 3$
$= 7$	$= -4 + 3$
	$= -1$

Left Side > Right Side

Try (10, 8).

Left Side	Right Side
$y$	$-x + 3$
$= 8$	$= -10 + 3$
	$= -7$

Left Side > Right Side

The ordered pairs (1, 0) and (-2, 1) are not solutions to the inequality  $y > -x + 1$ .

**b)**  $x + y \geq 6$

Try (2, 4).

Left Side	Right Side
$x + y$	6
$= 2 + 4$	
$= 6$	

Left Side = Right Side

Try (-5, 8).

Left Side	Right Side
$x + y$	6
$= (-5) + 8$	
$= 3$	

Left Side  $\neq$  Right Side

Try (4, 1).

Left Side	Right Side
$x + y$	6
$= 4 + 1$	
$= 5$	

Left Side  $\neq$  Right Side

Try (8, 2).

Left Side	Right Side
$x + y$	6
$= 8 + 2$	
$= 10$	

Left Side  $\geq$  Right Side

The ordered pairs (-5, 8) and (4, 1) are not solutions to the inequality  $x + y \geq 6$ .

**c)**  $4x - 3y < 10$

Try (1, 3).

Left Side	Right Side
$4x - 3y$	10
$= 4(1) - 3(3)$	
$= -5$	

Left Side < Right Side

Try (5, 1).

Left Side	Right Side
$4x - 3y$	10
$= 4(5) - 3(1)$	
$= 17$	

Left Side  $\not<$  Right Side

Try (-2, -3).

Left Side	Right Side
$4x - 3y$	10
$= 4(-2) - 3(-3)$	
$= 1$	

Left Side < Right Side

Try (5, 6).

Left Side	Right Side
$4x - 3y$	10
$= 4(5) - 3(6)$	
$= 2$	

Left Side < Right Side

The ordered pair (5, 1) is not a solution to the inequality  $4x - 3y < 10$ .

**d)**  $5x + 2y \leq 9$

Try (0, 0).

Left Side	Right Side
-----------	------------

$$5x + 2y$$

$$9$$

$$= 5(\textcolor{red}{0}) + 2(\textcolor{red}{0})$$

$$= 0$$

Left Side  $\leq$  Right Side

Try (3, -1).

Left Side	Right Side
-----------	------------

$$5x + 2y$$

$$9$$

$$= 5(\textcolor{red}{3}) + 2(\textcolor{red}{-1})$$

$$= 13$$

Left Side  $\not\leq$  Right Side

Try (-4, 2).

Left Side	Right Side
-----------	------------

$$5x + 2y$$

$$9$$

$$= 5(\textcolor{red}{-4}) + 2(\textcolor{red}{2})$$

$$= -16$$

Left Side  $\leq$  Right Side

Try (1, -2).

Left Side	Right Side
-----------	------------

$$5x + 2y$$

$$9$$

$$= 5(\textcolor{red}{1}) + 2(\textcolor{red}{-2})$$

$$= 1$$

Left Side  $\leq$  Right Side

The ordered pair (3, -1) is not a solution to the inequality  $5x + 2y \leq 9$ .

### Section 9.1 Page 472 Question 3

**a)**  $y \leq x + 3$

The equation is in the  $y = mx + b$  form.

The slope is 1 and the y-intercept is 3.

The boundary should be a solid line because  $y = x + 3$  is included.

**b)**  $y > 3x + 5$

The equation is in the  $y = mx + b$  form.

The slope is 3 and the y-intercept is 5.

The boundary should be a dashed line because  $y = 3x + 5$  is not included.

**c)**  $4x + y > 7$

Express in the  $y = mx + b$  form.

$$y > -4x + 7$$

The slope is -4 and the y-intercept is 7.

The boundary should be a dashed line because  $4x + y = 7$  is not included.

**d)**  $2x - y \leq 10$

Express in the  $y = mx + b$  form.

$$2x - 10 \leq y \text{ or } y \geq 2x - 10$$

The slope is 2 and the y-intercept is -10.

The boundary should be a solid line because  $2x - y = 10$  is included.

e)  $4x + 5y \geq 20$

$$5y \geq -4x + 20$$

$$y \geq -\frac{4}{5}x + 4$$

The slope is  $-\frac{4}{5}$  and the y-intercept is 4.

The boundary should be a solid line because  $4x + 5y = 20$  is included.

f)  $x - 2y < 10$

$$x - 10 < 2y$$

$$y > \frac{1}{2}x - 5$$

The slope is  $\frac{1}{2}$  and the y-intercept is  $-5$ .

The boundary should be a dashed line because  $x - 2y = 10$  is not included.

## Section 9.1 Page 472 Question 4

a)  $y \leq -2x + 5$

The slope is  $-2$  and the y-intercept is 5.

The x-intercept is 2.5.

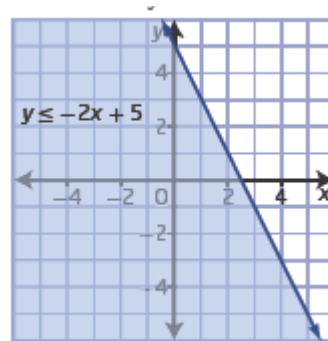
Use a solid line for the boundary, because  $y = -2x + 5$  is included.

Verify that the region to shade is below the line. Try (0, 0).

Left Side	Right Side
$= 0$	$= -2(0) + 5$
	$= 5$

$$\text{Left Side} \leq \text{Right Side}$$

The graph of the solution region is correct.



b)  $3y - x > 8$

$$3y > x + 8$$

$$y > \frac{1}{3}x + \frac{8}{3}$$

The slope is  $\frac{1}{3}$  and the y-intercept is  $\frac{8}{3}$ .

The x-intercept is  $-8$ .

Use a dashed line for the boundary, because  $3y - x = 8$  is not included.

Verify that the region to shade is above the line.

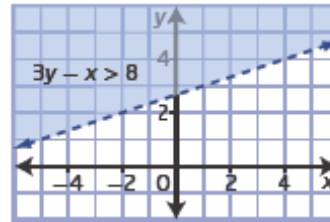
Try (0, 4).

Left Side                  Right Side

$$= 3(4) - 0 = 8$$

$$= 12$$

Left Side > Right Side



The graph of the solution region is correct.

**c)**  $4x + 2y - 12 \geq 0$

$$2y \geq -4x + 12$$

$$y \geq -2x + 6$$

The slope is  $-2$  and the  $y$ -intercept is  $6$ .

The  $x$ -intercept is  $3$ .

Use a solid line for the boundary, because

$$4x + 2y - 12 = 0 \text{ is included.}$$

Verify that the region to shade is above the line. Try (4, 0).

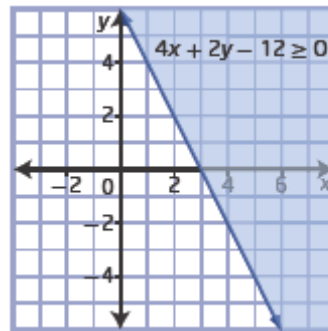
Left Side                  Right Side

$$= 4(4) + 2(0) - 12 = 4$$

$$= 4$$

Left Side > Right Side

The graph of the solution region is correct.



**d)**  $4x - 10y < 40$

$$4x - 40 < 10y$$

$$y > 0.4x - 4$$

The slope is  $0.4$  and the  $y$ -intercept is  $-4$ .

The  $x$ -intercept is  $10$ .

Use a dashed line for the boundary, because

$$4x - 10y = 40 \text{ is not included.}$$

Verify that the region to shade is above the line.

Try (0, 0).

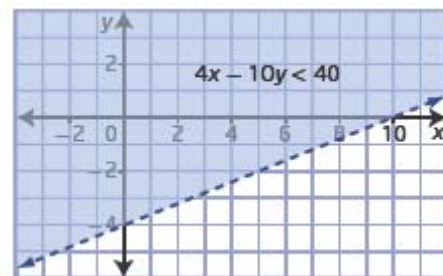
Left Side                  Right Side

$$= 4(0) - 10(0) = 0$$

$$= 40$$

Left Side < Right Side

The graph of the solution region is correct.



e)  $x \geq y - 6$

$x + 6 \geq y$  or  $y \leq x + 6$

The slope is 1 and the y-intercept is 6.

The x-intercept is -6.

Use a solid line for the boundary, because

$x = y - 6$  is included.

Verify that the region to shade is below the line.

Try (0, 0).

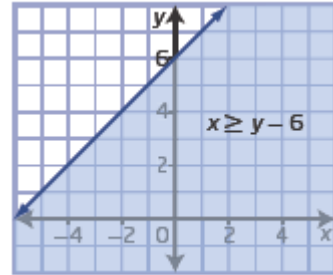
Left Side      Right Side

$= 0$                $= 0 - 6$

$= -6$

Left Side > Right Side

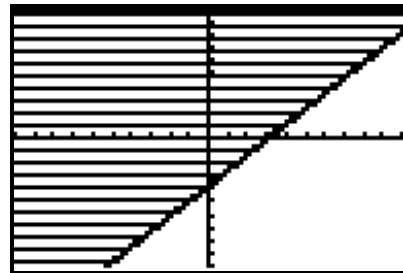
The graph of the solution region is correct.



**Section 9.1    Page 472      Question 5**

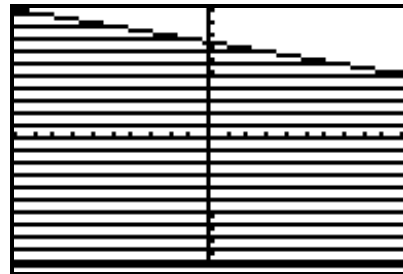
a)  $6x - 5y \leq 18$

$\frac{6}{5}x - \frac{18}{5} \leq y$  or  $y \geq \frac{6}{5}x - \frac{18}{5}$



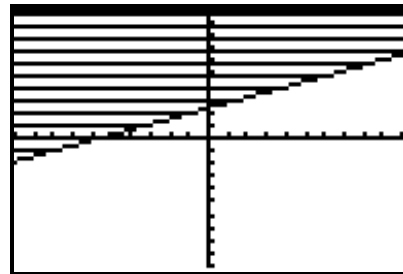
b)  $x + 4y < 30$

$y < -\frac{1}{4}x + \frac{15}{2}$



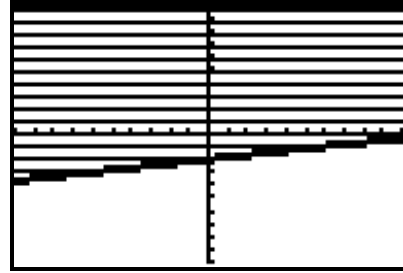
c)  $-5x + 12y - 28 > 0$

$y > \frac{5}{12}x + \frac{7}{3}$



d)  $x \leq 6y + 11$

$$\frac{1}{6}x - \frac{11}{6} \leq y \quad \text{or} \quad y \geq \frac{1}{6}x - \frac{11}{6}$$



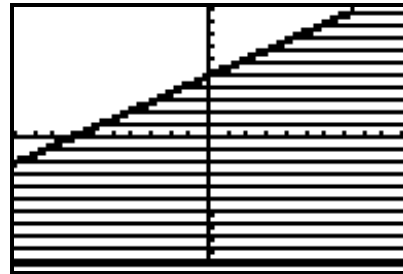
e)

$$3.6x - 5.3y + 30 \geq 4$$

$$3.6x + 30 - 4 \geq 5.3y$$

$$3.6x + 26 \geq 5.3y$$

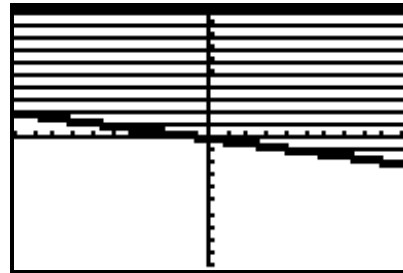
$$\frac{36}{53}x + \frac{260}{53} \geq y \quad \text{or} \quad y \leq \frac{36}{53}x + \frac{260}{53}$$



**Section 9.1 Page 472 Question 6**

$$-5y \leq x$$

$$y \geq -\frac{1}{5}x$$

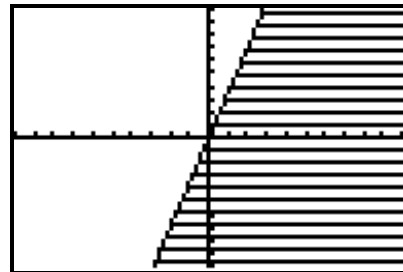


**Section 9.1 Page 472 Question 7**

$$7x - 2y > 0$$

$$7x > 2y$$

$$\frac{7}{2}x > y \quad \text{or} \quad y < \frac{7}{2}x$$



**Section 9.1 Page 472 Question 8**

a)  $6x + 3y \geq 21$

$$3y \geq -6x + 21$$

$$y \geq -2x + 7$$

From the equation, the y-intercept is 7 and the slope is  $-2$ , so this can be graphed by hand. The boundary  $y = -2x + 7$  is included, so use a solid line. Shade above the line.

Verify using (4, 0).

Left Side	Right Side
-----------	------------

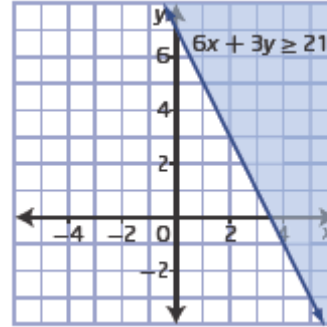
$6x + 3y$	21
-----------	----

$$= 6(4) + 3(0)$$

$$= 24$$

Left Side > Right Side

The correct region is shaded.



**b)**  $10x < 2.5y$

$$4x < y \text{ or } y > 4x$$

For the line  $y = 4x$ , the slope is 4 and the y-intercept is 0. This can be graphed by hand.

The boundary  $y = 4x$  is not included, so use a dashed line. Shade above the line.

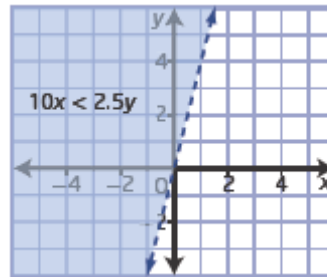
Test the point (0, 2).

Left Side	Right Side
-----------	------------

$10x$	$2.5y$
$= 10(0)$	$= 2.5(2)$
$= 0$	$= 5$

Left Side < Right Side

The correct region is shaded.



**c)**  $2.5x < 10y$

$$\frac{1}{4}x < y \text{ or } y > \frac{1}{4}x$$

The boundary line has equation  $y = \frac{1}{4}x$ . Its slope is  $\frac{1}{4}$  and y-intercept is 0. This line can

be graphed by hand. The line is not included, so use a dashed line. Shade above the line.

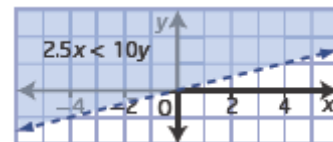
Test (0, 1).

Left Side	Right Side
-----------	------------

$2.5x$	$10y$
$= 2.5(0)$	$= 10(1)$
$= 0$	$= 10$

Left Side < Right Side

The correct region is shaded.

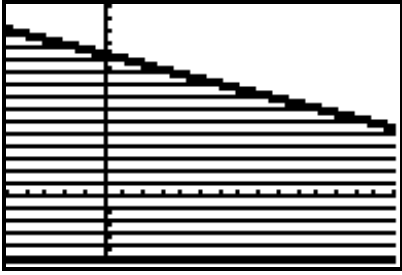


**d)**  $4.89x + 12.79y \leq 145$

$$12.79y \leq -4.89x + 145$$

$$y \leq -\frac{489}{1279}x + \frac{14500}{1279}$$

Since the numbers are not nice, use a graphing calculator.



e)  $0.8x - 0.4y > 0$

$$8x > 4y$$

$$2x > y \text{ or } y < 2x$$

The boundary is the line  $y = 2x$ . Its slope is 2 and y-intercept is 0. This can be graphed by hand. The line is not included, so use a dashed line. Shade below the line.

Test (1, 0).

Left Side	Right Side
-----------	------------

$$0.8x - 0.4y$$

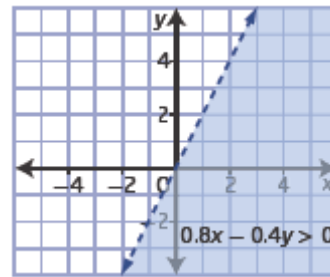
$$0$$

$$= 0.8(1) - 0.4(0)$$

$$= 0.8$$

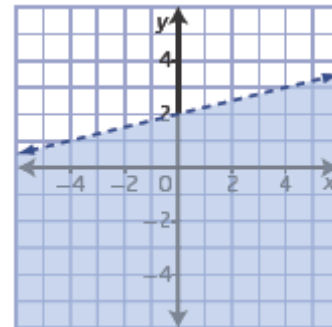
Left Side > Right Side

The correct region is shaded.

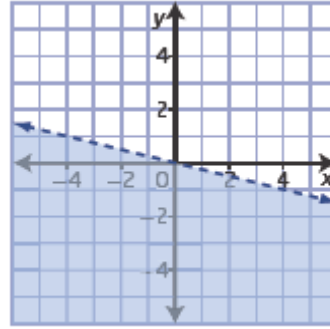


## Section 9.1 Page 472 Question 9

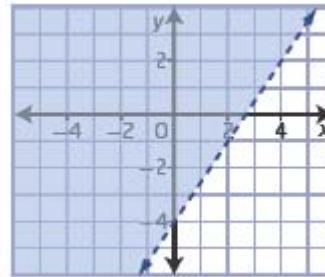
a) From the graph, the y-intercept of the boundary line is 2 and the slope is  $\frac{1}{4}$ . So the equation of the boundary line is  $y = \frac{1}{4}x + 2$ . Since the region shaded is below a dashed line, the inequality shown is  $y < \frac{1}{4}x + 2$ .



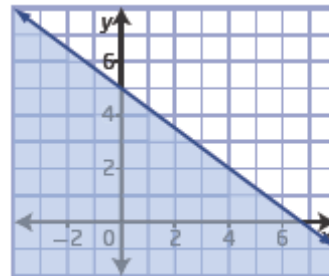
**b)** From the graph, the y-intercept of the boundary line is 0 and the slope is  $-\frac{1}{4}$ . So the equation of the boundary line is  $y = -\frac{1}{4}x$ . Since the region shaded is below a dashed line, the inequality shown is  $y < -\frac{1}{4}x$ .



**c)** From the graph, the y-intercept of the boundary line is  $-4$  and the slope is  $\frac{3}{2}$ . So the equation of the boundary line is  $y = \frac{3}{2}x - 4$ . Since the region shaded is above a dashed line, the inequality shown is  $y > \frac{3}{2}x - 4$ .



**d)** From the graph, the y-intercept of the boundary line is 5 and the slope is  $-\frac{3}{4}$ . So the equation of the boundary line is  $y = -\frac{3}{4}x + 5$ . Since the region shaded is below a solid line, the inequality shown is  $y \leq -\frac{3}{4}x + 5$ .

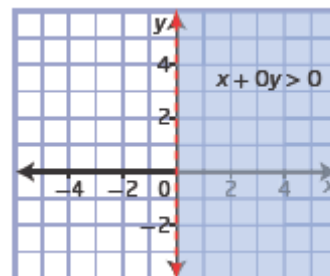


## Section 9.1 Page 473 Question 10

$$x + 0y > 0$$

The inequality simplifies to  $x > 0$ .

The line  $x = 0$  is the y-axis, so all of the plane to the right of the y-axis is the region where  $x > 0$ .



**Section 9.1 Page 473 Question 11**

**a)** Let  $x$  represent the number of hours Amaruq works and let  $y$  represent the number of pairs of baby moccasins she sells. If she wants to earn at least \$250, then  $12x + 12y \geq 250$ , where  $x \geq 0$  and  $y \geq 0$ .

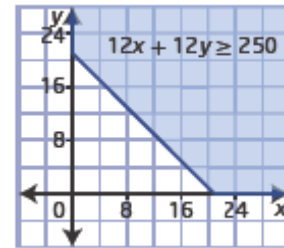
**b)** Rearrange the equation of the boundary line  $12x + 12y = 250$ , to the  $y = mx + b$  form.  
 $12y = -12x + 250$

$$y = -x + \frac{250}{12}$$

$$y = -x + 20\frac{5}{6}$$

So, the slope of the line is  $-1$  and the  $y$ -intercept is  $20\frac{5}{6}$ . Because the slope is  $-1$ , the  $x$ -intercept must also be  $20\frac{5}{6}$ . Use the two intercepts to graph the boundary.

Shade the region above the line and in the first quadrant because only positive values make sense in the context.



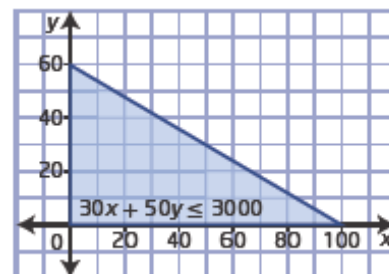
**c)** Examples: (24, 4) or 24 h work and sells 4 pairs of moccasins, (8, 16) or 8 h worked and sells 16 pairs of moccasins, (25, 0) or works 25 h and sells no moccasins.

**d)** Amaruq may have some weeks when she is not able to sell any moccasins, so her part-time job will provide some steady guaranteed income.

**Section 9.1 Page 473 Question 12**

**a)** Let  $x$  represent the number of hours Camille works with the elder and let  $y$  represent the number of hours of marketing assistance. If she wants to spend at most \$3000, then  $30x + 50y \leq 3000$ , where  $x \geq 0$  and  $y \geq 0$ .

**b)** From the equation of the boundary line  $30x + 50y = 3000$ , the  $x$ -intercept is at (110, 0) and the  $y$ -intercept is at (0, 60). Use the two intercepts to graph the boundary. Shade the region below the line and in the first quadrant because only positive values make sense in the context.



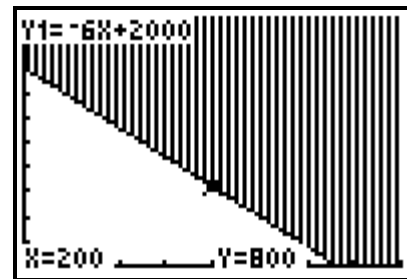
**Section 9.1 Page 473 Question 13**

Let  $x$  represent the number of minutes of phone use and  $y$  represent the number of megabytes of data use.

Then, Mariya's total cost for the phone each month is  $0.30x + 0.05y$ . To determine when this total cost is greater than \$100/month, graph the inequality  $0.30x + 0.05y > 100$ , where  $x \geq 0$  and  $y \geq 0$ .

From the equation of the boundary line  $0.30x + 0.05y = 100$ , or  $y = -6x + 2000$ , the slope is  $-6$  and the  $y$ -intercept is 2000.

The \$100 plan is a better choice if Mariya uses, for example, more than 334 min and no data per month, or 0 min of phone use and 2001 megabytes of data or any other combinations that are above the boundary line.

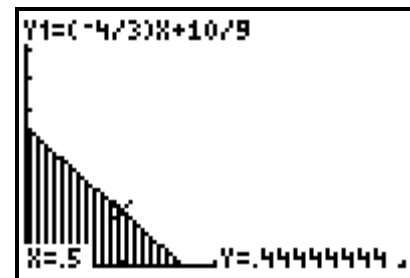


**Section 9.1 Page 474 Question 14**

Let  $x$  represent the area of glass, in square metres, and  $y$  represent the mass of nanomaterial used, in kilograms. Since the budget is still \$50, any amounts of materials in the region defined by  $60x + 45y \leq 50$ , where  $x \geq 0$  and  $y \geq 0$ .

Graph the region  $y \leq -\frac{4}{3}x + \frac{10}{9}$ .

Example: 0.5 m<sup>2</sup> of glass and 0.4 kg of nanomaterial.



**Section 9.1 Page 474 Question 15**

Let  $x$  represent the number of hours of ice rental and  $y$  represent the number of hours of gym rental. Then, the following inequality represents possible combinations that are within a budget of \$7000.

$125x + 55y \leq 7000$ , where  $x \geq 0$  and  $y \geq 0$ .

Graph the region  $y \leq -\frac{25}{11}x + \frac{1400}{11}$ .

Examples of possible times are:

12 h ice time and 100 h of gym time, or 23 h of ice time and 75 h of gym time.



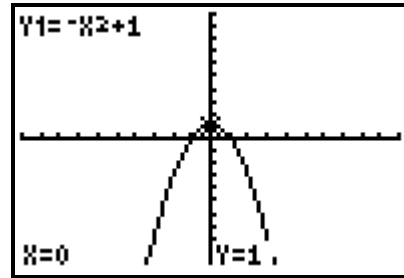
**Section 9.1 Page 474 Question 16**

Example:

a)  $y = -x^2 + 1$

b)  $y > -x^2 + 1$  describes the area of the plane that is above the parabola.

$y < -x^2 + 1$  describes the area of the plane that is below the parabola.



c) Yes, this does satisfy the definition of a solution region, the only difference is that the boundary is a parabola.

**Section 9.1 Page 474 Question 17**

For the upper left corner, determine the equation of the line joining (0, 384) and (512, 768). First determine the slope,  $m$ .

$$m = \frac{768 - 384}{512 - 0}$$

$$m = \frac{384}{512}$$

$$m = \frac{3}{4}$$

The equation for this boundary line is  $y = \frac{3}{4}x + 384$ . So, the inequality that represents

this region is  $y \geq \frac{3}{4}x + 384, 0 \leq x \leq 512$ .

For the lower left corner, determine the equation of the line joining (0, 384) and (512, 0).

$$m = \frac{0 - 384}{512 - 0}$$

$$m = \frac{-384}{512}$$

$$m = -\frac{3}{4}$$

The equation for this boundary line is  $y = -\frac{3}{4}x + 384$ . So, the inequality that represents

this region is  $y \leq -\frac{3}{4}x + 384, 0 \leq x \leq 512$ .

For the upper right corner, determine the equation of the line joining (512, 768) and (1024, 384).

$$m = \frac{768 - 384}{512 - 1024}$$

$$m = \frac{384}{-512}$$

$$m = -\frac{3}{4}$$

The equation is  $y = -\frac{3}{4}x + b$ . Substitute (512, 768) to determine  $b$ .

$$y = -\frac{3}{4}x + b$$

$$768 = -\frac{3}{4}(512) + b$$

$$768 + 384 = b$$

$$1152 = b$$

The equation for this boundary line is  $y = -\frac{3}{4}x + 1152$ . So, the inequality that represents

this region is  $y \geq -\frac{3}{4}x + 1152, 512 \leq x \leq 1024$ .

For the lower right corner, determine the equation of the line joining (512, 0) and (1024, 384).

$$m = \frac{0 - 384}{512 - 1024}$$

$$m = \frac{-384}{-512}$$

$$m = \frac{3}{4}$$

The equation is  $y = \frac{3}{4}x + b$ . Substitute (512, 0) to determine  $b$ .

$$y = \frac{3}{4}x + b$$

$$0 = \frac{3}{4}(512) + b$$

$$-384 = b$$

The equation for this boundary line is  $y = \frac{3}{4}x - 384$ . So, the inequality that represents this region is  $y \geq \frac{3}{4}x - 384$ ,  $512 \leq x \leq 1024$ .

## Section 9.1 Page 474 Question 18

**Step 1** The total budget for power is \$35 000/h, so the inequality that represent the cost of hydroelectric and wind power is:

$60x + 90y \leq 35\,000$ , where  $0 \leq x \leq 500$  and  $y \geq 0$ .

**Step 2** Graph  $y \leq -\frac{2}{3}x + \frac{3500}{9}$ , with  $0 \leq x \leq 500$  and  $y \geq 0$ .



For the boundary line,  $60x + 90y = 35\,000$ , the y-intercept is at approximately (388.9, 0), the right limit is (500, 55.6) and the lower bounds of the region are (0, 0) and (500, 0). The last value is because the agreement with Manitoba Hydro is for up to 500 MWh of hydroelectric power.

**Step 3** Use the revenue equation,  $R = 95x + 105y$  to determine the revenue for varied amounts of the two types of power. There are too many possible combinations of hydro and wind power to ensure that the maximum revenue is shown in the spreadsheet.

	A	B	C
1	Hydroelectric (MWh)	Wind (MWh)	Revenue (\$)
2	0	0	0
3	50	25	7375
4	100	50	14750
5	150	75	22125
6	200	100	29500
7			
8			

**Step 4** Check the upper vertices from step 2 in the revenue equation.

For (0, 388.9)

For (500, 55.6)

$R = 95(0) + 105(388.9)$

$R = 95(500) + 105(55.6)$

$R = 40\,834.5$

$R = 53\,338.00$

The maximum revenue is \$53 338, when there is 500 MWh of hydroelectric power and approximately 55.6 MWh of wind power.

**Section 9.1 Page 475 Question 19**

Answers may vary. Example:

	Example 1	Example 2	Example 3	Example 4
Give an example for each type of linear inequality.	$y \geq x - 1$	$y \leq x - 1$	$y > x - 1$	$y < x - 1$
State the inequality sign.	$\geq$ , greater than or equal to	$\leq$ , less than or equal to	$>$ , greater than	$<$ , less than
Boundary solid or broken?	solid	solid	broken	broken
Which region do you shade?	above	below	above	below

**Section 9.1 Page 475 Question 20**

Answers will vary.

The boundary line is  $y = -\frac{3}{5}x + 30$  or  $3x + 5y = 150$ , so the region is  $3x + 5y \leq 150$ ,

where  $x \geq 0$  and  $y \geq 0$ . A scenario is as follows.

Jackie has \$150 to buy treats for her friends. If ice-cream cones cost \$3 each and smoothies cost \$5 each, what combinations of the treats can she buy?

**Section 9.1 Page 475 Question 21**

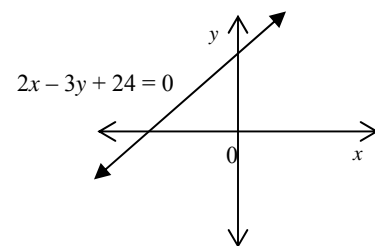
**a)** Use a sketch graph to help visualize the situation.

For the boundary line,  $2x - 3y + 24 = 0$ , the

$x$ -intercept is  $-12$  and the  $y$ -intercept is  $8$ .

So the area of the triangular region bounded by the line  $2x - 3y + 24 = 0$  and the axes is

$\frac{1}{2}(8)(12)$  or 48 square units.



**b)** The  $y$ -intercept tells the height of the triangular region.

**c)** The slope affects the  $x$ -intercept, which gives the base of the triangular region.

**d)** The area would not change because, even if either of the intercepts is negative, the distance from the origin is still the same positive quantity.

## Section 9.2 Quadratic Inequalities in One Variable

### Section 9.2 Page 484 Question 1

a)  $x^2 - 4x + 3 \leq 0$

$$(x - 3)(x - 1) \leq 0$$

The factors confirm that  $f(x)$  intersects the  $x$ -axis at 1 and 3.

The solution for  $f(x) \leq 0$  is  $\{x \mid 1 \leq x \leq 3, x \in \mathbb{R}\}$ .

b)  $x^2 - 4x + 3 \geq 0$

$$(x - 3)(x - 1) \geq 0$$

The factors confirm that  $f(x)$  intersects the  $x$ -axis at 1 and 3.

The solution for  $f(x) \geq 0$  is  $\{x \mid x \leq 1 \text{ or } x \geq 3, x \in \mathbb{R}\}$ .

c)  $x^2 - 4x + 3 > 0$

$$(x - 3)(x - 1) > 0$$

The factors confirm that  $f(x)$  intersects the  $x$ -axis at 1 and 3.

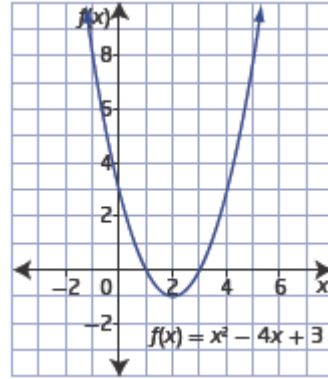
The solution for  $f(x) > 0$  is  $\{x \mid x < 1 \text{ or } x > 3, x \in \mathbb{R}\}$ .

d)  $x^2 - 4x + 3 < 0$

$$(x - 3)(x - 1) < 0$$

The factors confirm that  $f(x)$  intersects the  $x$ -axis at 1 and 3.

The solution for  $f(x) < 0$  is  $\{x \mid 1 < x < 3, x \in \mathbb{R}\}$ .



### Section 9.2 Page 484 Question 2

Determine the roots of  $g(x) = -x^2 + 4x - 4$ .

$$0 = -(x^2 - 4x + 4)$$

$$0 = -(x - 2)^2$$

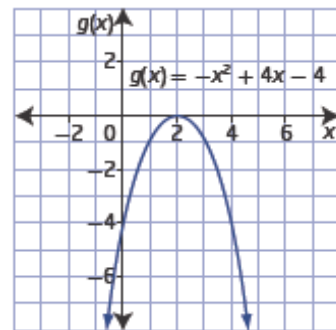
This confirms that the single root of  $g(x)$  is 2.

a) The solution for  $g(x) \leq 0$  is  $\{x \mid x \in \mathbb{R}\}$ .

b) The solution for  $g(x) \geq 0$  is  $\{x \mid x = 2, x \in \mathbb{R}\}$ .

c)  $g(x) > 0$  has no solution.

d) The solution for  $g(x) < 0$  is  $\{x \mid x \neq 2, x \in \mathbb{R}\}$ .



**Section 9.2    Page 484    Question 3**

**a)** Substitute  $x = 4$  into  $x^2 - 3x - 10$ .

$$\begin{aligned} & 4^2 - 3(4) - 10 \\ &= 16 - 12 - 10 \\ &= -6 \end{aligned}$$

Since  $-6 \not> 0$ ,  $x = 4$  is not a solution to  $x^2 - 3x - 10 > 0$ .

**b)** Substitute  $x = 1$  into  $x^2 + 3x - 4$ .

$$\begin{aligned} & 1^2 + 3(1) - 4 \\ &= 0 \end{aligned}$$

So  $x = 1$  is a solution to  $x^2 + 3x - 4 \geq 0$ .

**c)** Substitute  $x = -2$  into  $x^2 + 4x + 3$ .

$$\begin{aligned} & (-2)^2 + 4(-2) + 3 \\ &= 4 - 8 + 3 \\ &= -1 \end{aligned}$$

Since  $-1 < 0$ ,  $x = -2$  is a solution to  $x^2 + 4x + 3 < 0$ .

**d)** Substitute  $x = -3$  into  $-x^2 - 5x - 4$ .

$$\begin{aligned} & -(-3)^2 - 5(-3) - 4 \\ &= -9 + 15 - 4 \\ &= 2 \end{aligned}$$

Since  $2 > 0$ ,  $x = -2$  is not a solution to  $-x^2 - 5x - 4 \leq 0$ .

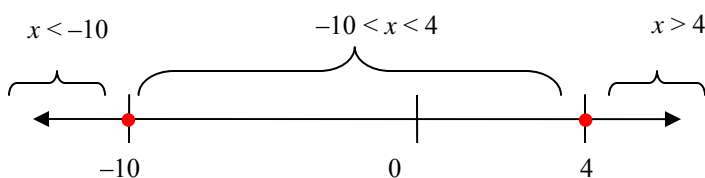
**Section 9.2    Page 485    Question 4**

**a)** Solve  $x(x + 6) \geq 40$ .

First, determine the roots of the related equation.

$$\begin{aligned} & x(x + 6) = 40 \\ & x^2 + 6x - 40 = 0 \\ & (x + 10)(x - 4) = 0 \\ & x + 10 = 0 \text{ or } x - 4 = 0 \\ & x = -10 \text{ or } x = 4 \end{aligned}$$

Plot  $-10$  and  $4$  on a number line.



Choose test points in each of the three intervals as shown in the table.

Interval	$x < -10$	$-10 < x < 4$	$x > 4$
Test Point	-12	0	5
Substitution	$-12(-12 + 6)$ $= -12(-6)$ $= 72$	$0(0 + 6)$ $= 0$	$5(5 + 6)$ $= 5(11)$ $= 55$
Is $x(x + 6) \geq 40$ ?	yes	no	yes

The solution set is  $\{x \mid x \leq -10 \text{ or } x \geq 4, x \in \mathbb{R}\}$ .

b) Solve  $-x^2 - 14x - 24 < 0$ .

First, determine the roots of the related equation.

$$-x^2 - 14x - 24 = 0$$

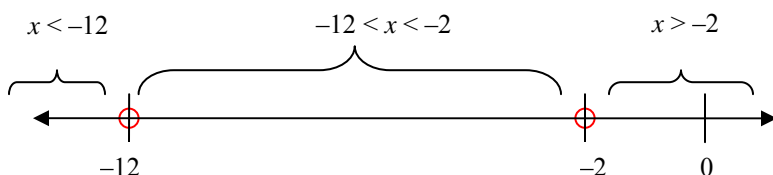
$$x^2 + 14x + 24 = 0$$

$$(x + 12)(x + 2) = 0$$

$$x + 12 = 0 \text{ or } x + 2 = 0$$

$$x = -12 \text{ or } x = -2$$

Plot  $-12$  and  $-2$  on a number line.



Choose test points in each of the three intervals as shown in the table.

Interval	$x < -12$	$-12 < x < -2$	$x > -2$
Test Point	-20	-5	0
Substitution	$-(-20)^2 - 14(-20) - 24$ $= -400 - 280 - 24$ $= -704$	$-(-5)^2 - 14(-5) - 24$ $= 25 + 70 - 24$ $= 71$	$-(0)^2 - 14(0) - 24$ $= -24$
Is $-x^2 - 14x - 24 < 0$ ?	yes	no	yes

The solution set is  $\{x \mid x < -12 \text{ or } x > -2, x \in \mathbb{R}\}$ .

c) Solve  $6x^2 > 11x + 35$ .

First, determine the roots of the related equation.

$$6x^2 = 11x + 35$$

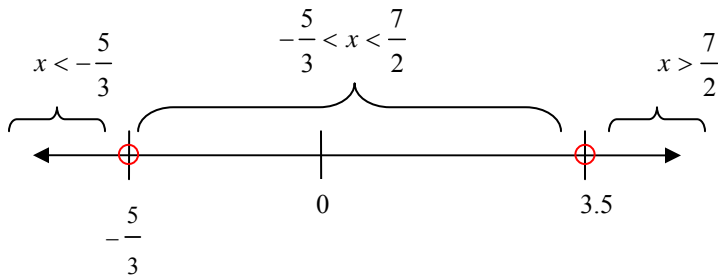
$$6x^2 - 11x - 35 = 0$$

$$(3x + 5)(2x - 7) = 0$$

$$3x + 5 = 0 \text{ or } 2x - 7 = 0$$

$$x = -\frac{5}{3} \text{ or } x = \frac{7}{2}$$

Plot  $-\frac{5}{3}$  and  $\frac{7}{2}$  on a number line.



Choose test points in each of the three intervals as shown in the table.

Interval	$x < -\frac{5}{3}$	$-\frac{5}{3} < x < \frac{7}{2}$	$x > \frac{7}{2}$
Test Point	-2	0	5
Substitution	$6x^2 = 6(-2)^2$ $= 24$ $11x + 35 = 11(-2) + 35$ $= 13$	$6x^2 = 6(0)^2$ $= 0$ $11x + 35 = 11(0) + 35$ $= 35$	$6x^2 = 6(5)^2$ $= 150$ $11x + 35 = 11(5) + 35$ $= 90$
Is $6x^2 > 11x + 35$ ?	yes	no	yes

The solution set is  $\{x \mid x < -\frac{5}{3} \text{ or } x > \frac{7}{2}, x \in \mathbb{R}\}$ .

d) Solve  $8x + 5 \leq -2x^2$ .

First, determine the roots of the related equation.

$$8x + 5 = -2x^2$$

$$2x^2 + 8x + 5 = 0$$

The equation does not factor. Use the quadratic formula with  $a = 2$ ,  $b = 8$ , and  $c = 5$ .

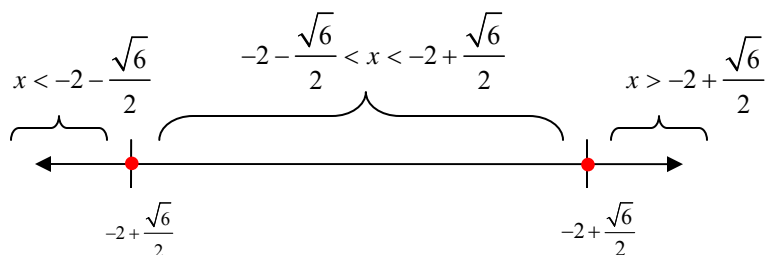
$$x = \frac{-8 \pm \sqrt{8^2 - 4(2)(5)}}{2(2)}$$

$$x = \frac{-8 \pm \sqrt{24}}{4}$$

$$x = -2 \pm \frac{\sqrt{6}}{2}$$

Plot  $-2 + \frac{\sqrt{6}}{2}$  and  $-2 - \frac{\sqrt{6}}{2}$  on a number line.

The points are approximately  $-0.775$  and  $-3.22$ .



Choose test points in each of the three intervals as shown in the table.

Interval	$x < -2 - \frac{\sqrt{6}}{2}$	$-2 - \frac{\sqrt{6}}{2} < x < -2 + \frac{\sqrt{6}}{2}$	$x > -2 + \frac{\sqrt{6}}{2}$
Test Point	$-4$	$-1$	$0$
Substitution	$8x + 5 = 8(-4) + 5$ $= -27$ $-2x^2 = -2(-4)^2$ $= -32$	$8x + 5 = 8(-1) + 5$ $= -3$ $-2x^2 = -2(-1)^2$ $= -2$	$8x + 5 = 8(0) + 5$ $= 5$ $-2x^2 = -2(0)^2$ $= 0$
Is $8x + 5 \leq -2x^2$ ?	no	yes	no

The solution set is  $\{x \mid -2 - \frac{\sqrt{6}}{2} \leq x \leq -2 + \frac{\sqrt{6}}{2}, x \in \mathbb{R}\}$ .

## Section 9.2 Page 485 Question 5

a)  $x^2 + 3x \leq 18$

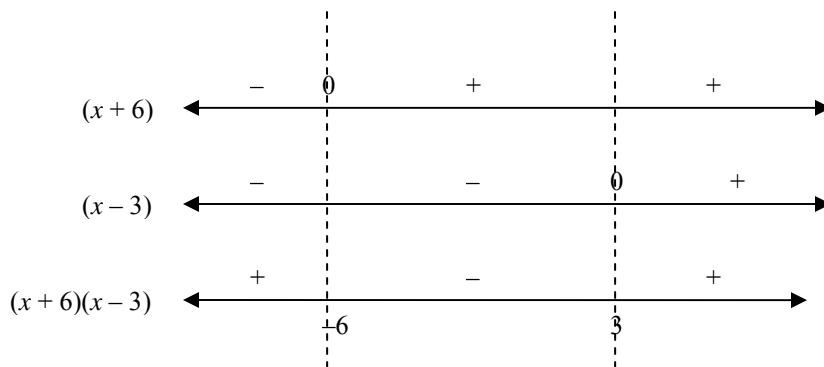
$$x^2 + 3x - 18 \leq 0$$

$$(x + 6)(x - 3) \leq 0$$

Substitute  $3$  in  $(x + 6)$ :  $3 + 6 = 9$  is positive.

Substitute  $-6$  in  $(x - 3)$ :  $-6 - 3 = -9$  is negative.

The signs of the factors in each interval are shown on the diagram below.



So, the middle interval is where  $x^2 + 3x - 18 \leq 0$  or  $x^2 + 3x \leq 18$ .

The solution set is  $\{x \mid -6 \leq x \leq 3, x \in \mathbf{R}\}$ .

**b)**  $x^2 + 3 \geq -4x$

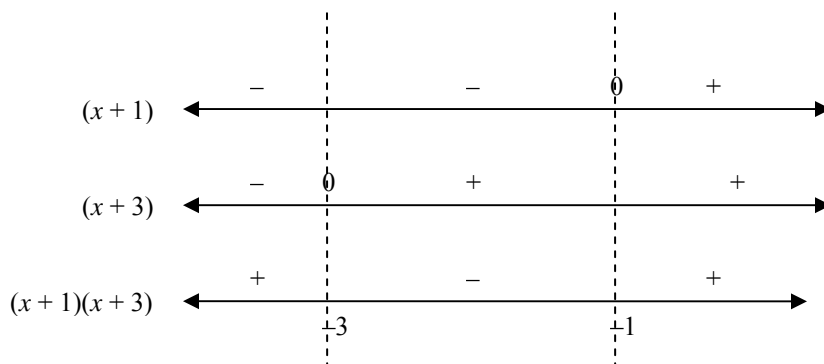
$$x^2 + 4x + 3 \geq 0$$

$$(x + 1)(x + 3) \geq 0$$

Substitute  $-1$  in  $(x + 3)$ :  $-1 + 3 = 2$  is positive.

Substitute  $-3$  in  $(x + 1)$ :  $-3 + 1 = -2$  is negative.

The signs of the factors in each interval are shown on the diagram below.



So, the two outer intervals are where  $x^2 + 4x + 3 \geq 0$  or  $x^2 + 3 \geq -4x$ .

The solution set is  $\{x \mid x \leq -3 \text{ or } x \geq -1, x \in \mathbf{R}\}$ .

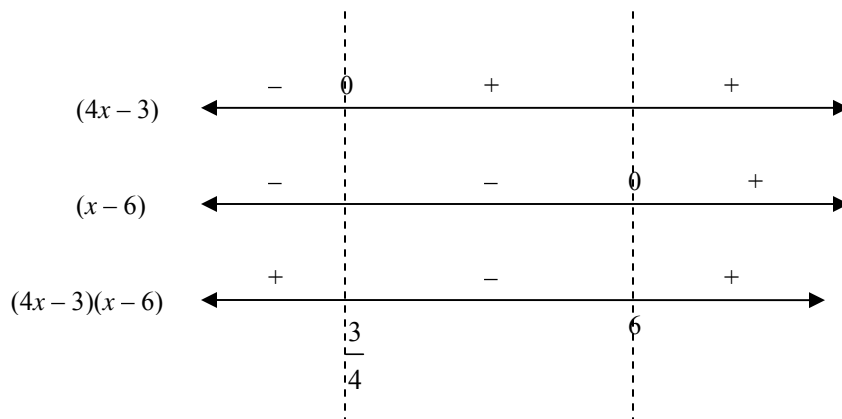
**c)**  $4x^2 - 27x + 18 < 0$

$$(4x - 3)(x - 6) < 0$$

Substitute  $\frac{3}{4}$  in  $(x - 6)$ :  $\frac{3}{4} - 6 = -\frac{21}{4}$  is negative.

Substitute  $6$  in  $(4x - 3)$ :  $4(6) - 3 = 21$  is positive.

The signs of the factors in each interval are shown on the diagram below.



So, the middle interval is where  $4x^2 - 27x + 18 < 0$ .

The solution set is  $\{x \mid \frac{3}{4} < x < 6, x \in \mathbb{R}\}$ .

**d)**  $-6x \geq x^2 - 16$

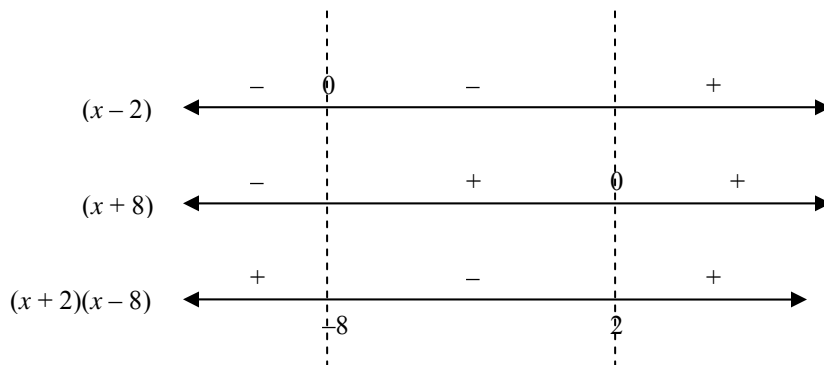
$$x^2 + 6x - 16 \leq 0$$

$$(x-2)(x+8) \leq 0$$

Substitute  $-8$  in  $(x-2)$ :  $-8 - 2 = -10$  is negative.

Substitute  $2$  in  $(x+8)$ :  $2 + 8 = 10$  is positive.

The signs of the factors in each interval are shown on the diagram below.



So, the middle interval is where  $x^2 + 6x - 16 \leq 0$  or  $-6x \geq x^2 - 16$ .

The solution set is  $\{x \mid -8 \leq x \leq 2, x \in \mathbb{R}\}$ .

## Section 9.2 Page 485 Question 6

**a)**  $x^2 - 2x - 15 < 0$

$$(x-5)(x+3) < 0$$

*Case 1:* The first factor is negative and the second factor is positive.

$$x - 5 < 0 \text{ and } x + 3 > 0$$

$$x < 5 \text{ and } x > -3$$

These two inequalities are true for all points between  $-3$  and  $5$ .

*Case 2:* The first factor is positive and the second factor is negative.

$$x - 5 > 0 \text{ and } x + 3 < 0$$

$$x > 5 \text{ and } x < -3$$

The pair of conditions is never true.

The solution set is  $\{x \mid -3 < x < 5, x \in \mathbb{R}\}$ .

**b)**  $x^2 + 13x > -12$

$$x^2 + 13x + 12 > 0$$

$$(x + 12)(x + 1) > 0$$

*Case 1:* Both factors are positive.

$$x + 12 > 0 \text{ and } x + 1 > 0$$

$$x > -12 \text{ and } x > -1$$

Both conditions are true when  $x > -1$ .

*Case 2:* Both factors are negative.

$$x + 12 < 0 \text{ and } x + 1 < 0$$

$$x < -12 \text{ and } x < -1$$

Both conditions are true when  $x < -12$ .

The solution set is  $\{x \mid x < -12 \text{ or } x > -1, x \in \mathbb{R}\}$ .

**c)**  $-x^2 + 2x + 5 \leq 0$

$$x^2 - 2x - 5 \geq 0$$

Use the quadratic formula to solve  $x^2 - 2x - 5 = 0$ .

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-5)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{24}}{2}$$

$$x = 1 \pm \sqrt{6}$$

So, the inequality has two factors  $(x - 1 - \sqrt{6})(x - 1 + \sqrt{6})$ .

For their product to be greater than or equal to zero, both factors must be the same sign.

*Case 1:* Both factors are positive.

$$x - 1 - \sqrt{6} \geq 0 \text{ and } x - 1 + \sqrt{6} \geq 0$$

$$x \geq 1 + \sqrt{6} \text{ and } x \geq 1 - \sqrt{6}$$

These two inequalities are both true when  $x \geq 1 + \sqrt{6}$ .

*Case 2:* Both factors are negative.

$$x - 1 - \sqrt{6} \leq 0 \text{ and } x - 1 + \sqrt{6} \leq 0$$

$$x \leq 1 + \sqrt{6} \quad \text{and} \quad x \leq 1 - \sqrt{6}$$

The pair of conditions is true when  $x \leq 1 - \sqrt{6}$ .

The solution set is  $\{x \mid x \leq 1 - \sqrt{6} \text{ or } x \geq 1 + \sqrt{6}, x \in \mathbb{R}\}$ .

**d)**  $2x^2 \geq 8 - 15x$

$$2x^2 + 15x - 8 \geq 0$$

$$(2x - 1)(x + 8) \geq 0$$

Case 1: Both factors are positive.

$$2x - 1 \geq 0 \text{ and } x + 8 \geq 0$$

$$x \geq \frac{1}{2} \text{ and } x \geq -8$$

Both conditions are true when  $x \geq \frac{1}{2}$ .

Case 2: Both factors are negative.

$$2x - 1 \leq 0 \text{ and } x + 8 \leq 0$$

$$x \leq \frac{1}{2} \text{ and } x \leq -8$$

Both conditions are true when  $x \leq -8$ .

The solution set is  $\{x \mid x \leq -8 \text{ or } x \geq \frac{1}{2}, x \in \mathbb{R}\}$ .

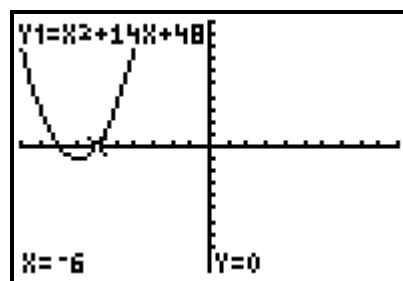
## Section 9.2 Page 485 Question 7

**a)**  $x^2 + 14x + 48 \leq 0$

Graph  $y = x^2 + 14x + 48$ .

The graph is below the  $x$ -axis between  $-8$  and  $-6$ .

The solution set is  $\{x \mid -8 \leq x \leq -6, x \in \mathbb{R}\}$ .



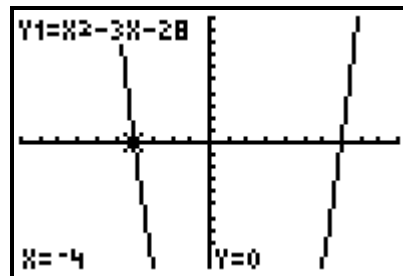
**b)**  $x^2 \geq 3x + 28$

$$x^2 - 3x - 28 \geq 0$$

Graph  $y = x^2 - 3x - 28$ .

The graph is above the  $x$ -axis when values of  $x$  are less than  $-4$  and greater than  $7$ .

The solution set is  $\{x \mid x \leq -4 \text{ or } x \geq 7, x \in \mathbb{R}\}$ .

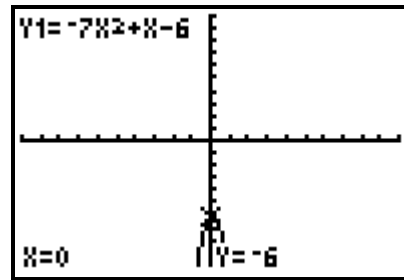


c)  $-7x^2 + x - 6 \geq 0$

Graph  $y = -7x^2 + x - 6$ .

The graph is never above the  $x$ -axis.

There is no solution for  $-7x^2 + x - 6 \geq 0$ .



d)  $4x(x - 1) > 63$

$4x^2 - 4x - 63 > 0$

Graph  $y = 4x^2 - 4x - 63$ .

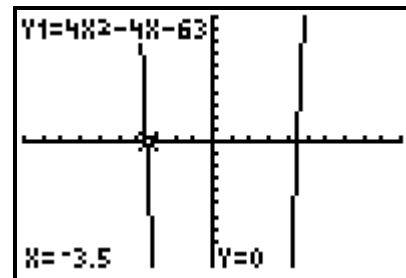
Factor to determine the zeros.

$0 = (2x - 9)(2x + 7)$

The zeros are 4.5 and  $-3.5$ .

The graph is above the  $x$ -axis to the left of  $-3.5$  and to the right of 4.5.

The solution set is  $\{x \mid x < -3.5 \text{ or } x > 4.5, x \in \mathbb{R}\}$ .



## Section 9.2 Page 485 Question 8

Methods may vary.

a)  $x^2 - 10x + 16 < 0$

Use case analysis because the expression is easy to factor.

$(x - 8)(x - 2) < 0$

Case 1: The first factor is negative and the second factor is positive.

$x - 8 < 0$  and  $x - 2 > 0$

$x < 8$  and  $x > 2$

These inequalities are true for values of  $x$  between 2 and 8.

Case 2: The first factor is positive and the second factor is negative.

$x - 8 > 0$  and  $x - 2 < 0$

$x > 8$  and  $x < 2$

Both inequalities are never true.

The solution set is  $\{x \mid 2 < x < 8, x \in \mathbb{R}\}$ .

b)  $12x^2 - 11x - 15 \geq 0$

Use case analysis because the expression can be factored.

$(3x - 5)(4x + 3) \geq 0$

Case 1: Both factors are positive.

$3x - 5 \geq 0$  and  $4x + 3 \geq 0$

$x \geq \frac{5}{3}$  and  $x \geq -\frac{3}{4}$

Both of these conditions are true when  $x \geq \frac{5}{3}$ .

*Case 1:* Both factors are negative.

$$3x - 5 \leq 0 \text{ and } 4x + 3 \leq 0$$

$$x \leq \frac{5}{3} \text{ and } x \leq -\frac{3}{4}$$

Both of these conditions are true when  $x \leq -\frac{3}{4}$ .

The solution set is  $\{x \mid x \leq -\frac{3}{4} \text{ or } x \geq \frac{5}{3}, x \in \mathbb{R}\}$ .

c)  $x^2 - 2x - 12 \leq 0$

Graph the function  $y = x^2 - 2x - 12$ .

Use the quadratic formula to find the roots.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

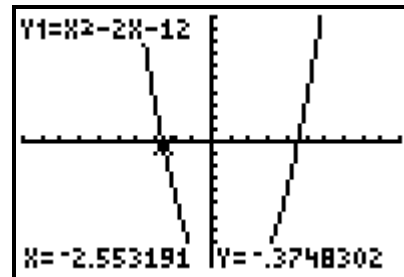
$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-12)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{52}}{2}$$

$$x = 1 \pm \sqrt{13}$$

The function is below the  $x$ -axis between the two roots.

The solution set is  $\{x \mid 1 - \sqrt{13} \leq x \leq 1 + \sqrt{13}, x \in \mathbb{R}\}$ .



d)  $x^2 - 6x + 9 > 0$

Use case analysis because the expression is easy to factor.

$$(x - 3)(x - 3) > 0$$

$$(x - 3)^2 > 0$$

This condition is true for all real values except  $x = 3$ .

The solution set is  $\{x \mid x \neq 3, x \in \mathbb{R}\}$ .

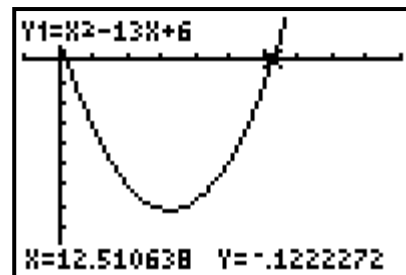
## Section 9.2 Page 485 Question 9

a)  $x^2 - 3x + 6 \leq 10x$

$$x^2 - 13x + 6 \leq 0$$

Graph  $y = x^2 - 13x + 6$  to check where the function is below the  $x$ -axis.

Use the quadratic formula to determine the roots.



$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-13) \pm \sqrt{(-13)^2 - 4(1)(6)}}{2(1)}$$

$$x = \frac{13 \pm \sqrt{145}}{2}$$

The solution set is  $\{x \mid \frac{13 - \sqrt{145}}{2} \leq x \leq \frac{13 + \sqrt{145}}{2}, x \in \mathbb{R}\}$ .

**b)**  $2x^2 + 12x - 11 > x^2 + 2x + 13$

$$x^2 + 10x - 24 > 0$$

$$(x + 12)(x - 2) > 0$$

*Case 1:* Both factors are positive.

$$x + 12 > 0 \text{ and } x - 2 > 0$$

$$x > -12 \text{ and } x > 2$$

Both conditions hold for  $x > 2$ .

*Case 2:* Both factors are negative.

$$x + 12 < 0 \text{ and } x - 2 < 0$$

$$x < -12 \text{ and } x < 2$$

Both conditions hold for  $x < -12$ .

The solution set is  $\{x \mid x < -12 \text{ or } x > 2, x \in \mathbb{R}\}$ .

**c)**  $x^2 - 5x < 3x^2 - 18x + 20$

$$0 < 2x^2 - 13x + 20$$

$$0 < (2x - 5)(x - 4)$$

*Case 1:* Both factors are positive.

$$2x - 5 > 0 \text{ and } x - 4 > 0$$

$$x > \frac{5}{2} \text{ and } x > 4$$

Both conditions hold for  $x > 4$ .

*Case 2:* Both factors are negative.

$$2x - 5 < 0 \text{ and } x - 4 < 0$$

$$x < \frac{5}{2} \text{ and } x < 4$$

Both conditions are true when  $x < \frac{5}{2}$ .

The solution set is  $\{x \mid x < \frac{5}{2} \text{ or } x > 4, x \in \mathbb{R}\}$ .

$$\begin{aligned}
 \text{d) } -3(x^2 + 4) &\leq 3x^2 - 5x - 68 \\
 -3x^2 - 12 &\leq 3x^2 - 5x - 68 \\
 0 &\leq 6x^2 - 5x - 56 \\
 0 &\leq (3x + 8)(2x - 7)
 \end{aligned}$$

Case 1: Both factors are positive.

$$3x + 8 \geq 0 \text{ and } 2x - 7 \geq 0$$

$$x \geq -\frac{8}{3} \text{ and } x \geq \frac{7}{2}$$

Both inequalities are true when  $x \geq \frac{7}{2}$ .

Case 2: Both factors are negative.

$$3x + 8 \leq 0 \text{ and } 2x - 7 \leq 0$$

$$x \leq -\frac{8}{3} \text{ and } x \leq \frac{7}{2}$$

Both inequalities are true when  $x \leq -\frac{8}{3}$ .

The solution set is  $\{x \mid x \leq -\frac{8}{3} \text{ or } x \geq \frac{7}{2}, x \in \mathbb{R}\}$ .

## Section 9.2 Page 485 Question 10

$$\begin{aligned}
 \text{a) } 9h^2 &\geq 750 \\
 h^2 &\geq \frac{750}{9}
 \end{aligned}$$

Since  $h$  represents thickness of ice, only the positive root has meaning in the context.

$$h \geq \frac{5\sqrt{30}}{3}.$$

Ice that is  $\frac{5\sqrt{30}}{3}$  cm (approximately 9.13 cm) or thicker will support the vehicle.

b) The inequality  $9h^2 \geq 1500$  can be used to find the thickness of ice that will support a mass of 1500 kg.

$$\text{c) } h^2 \geq \frac{1500}{9}$$

Since  $h$  represents thickness of ice, only the positive root has meaning in the context.

$$h \geq \frac{10\sqrt{15}}{3}.$$

Ice that is  $\frac{10\sqrt{15}}{3}$  cm (approximately 12.91 cm) or thicker will support the vehicle.

- d) The relationship between ice thickness and mass is quadratic, so the thickness does not double for double the mass. The thickness increases by a factor of  $\sqrt{2}$  when the mass doubles.

**Section 9.2   Page 486   Question 11**

- a) Let  $x$  represent the radius, in metres, of the circular area.

The area irrigated is 63 ha, or  $63(10\,000) \text{ m}^2$ .

Then the inequality that models the area that Murray can irrigate is  $\pi x^2 \leq 630\,000$ .

- b) Solve  $\pi x^2 \leq 630\,000$ . Since  $x$  represents a length, only positive roots need to be considered.

$$0 \leq x \leq \sqrt{\frac{630\,000}{\pi}} \text{ or } 0 \leq x \leq 100\sqrt{\frac{63}{\pi}}$$

- c) The radius is between 0 m and 447.81 m.

**Section 9.2   Page 486   Question 12**

- a) Substitute  $P = 10$  in  $-t^2 + 14 \leq P$ .

$$-t^2 + 14 \leq 10$$

$$4 \leq t^2$$

Since  $t$  represents years from the present, only the positive root needs to be considered.

$$t \geq 2$$

Carbon fibre will be \$10/kg or less, 2 years or more from now.

- b) Negative solutions for the inequality do not make sense in the context of time from now.

- c) Substitute  $P = 5$  in  $-t^2 + 14 \leq P$ .

$$-t^2 + 14 \leq 5$$

$$9 \leq t^2$$

$$t \geq 3$$

Carbon fibre will be \$5/kg or less, 3 years or more from now.

**Section 9.2    Page 486    Question 13**

Let  $x$  represent the length, in centimeters, of the shorter leg. Then,  $x + 2$  represents the length of the other leg.

$$\text{Area of triangle} = \frac{(\text{base})(\text{height})}{2}$$

$$\frac{x(x+2)}{2} \geq 4$$

$$x^2 + 2x \geq 8$$

$$x^2 + 2x - 8 \geq 0$$

$$(x+4)(x-2) \geq 0$$

*Case 1:* Both factors are positive.

$$x+4 \geq 0 \text{ and } x-2 \geq 0$$

$$x \geq -4 \text{ and } x \geq 2$$

These conditions are true for  $x \geq 2$ .

*Case 2:* Both factors are negative.

$$x+4 \leq 0 \text{ and } x-2 \leq 0$$

$$x \leq -4 \text{ and } x \leq 2$$

These conditions are true for  $x \leq -4$ . However, since  $x$  is a length, negative values do not make sense in this context.

The shorter side should be 2 cm or more to ensure that the area of the triangle is 4 cm<sup>2</sup> or more.

**Section 9.2    Page 486    Question 14**

**a)**  $ax^2 + bx + c \geq 0$

For the value of the quadratic to be greater than or equal to zero, the whole parabola must lie on or above the  $x$ -axis. So  $a > 0$  and the related quadratic equation has at most one  $x$ -intercept, when the vertex lies on the  $x$ -axis, otherwise the parabola is totally above the  $x$ -axis.

This occurs when the discriminant is less than equal to 0.

So all real numbers satisfy  $ax^2 + bx + c \geq 0$ , when  $a > 0$  and  $b^2 - 4ac \leq 0$ .

**b)** There is exactly one real solution to  $ax^2 + bx + c \geq 0$ , if the parabola opens downward but it has its vertex on the  $x$ -axis so at that  $x$ -value  $ax^2 + bx + c = 0$ . This occurs when  $a < 0$  and  $b^2 - 4ac = 0$ .

**c)** Provided  $a \neq 0$ , and  $b^2 - 4ac > 0$  then the related quadratic function  $ax^2 + bx + c = 0$  has two  $x$ -intercepts. In these cases the inequality  $ax^2 + bx + c \geq 0$  will be true for part of the domain only:

- outside the  $x$ -intercepts when  $a > 0$  because the vertex is below the  $x$ -axis and the parabola opens upward and
- between the  $x$ -intercepts when  $a < 0$  because the vertex is above the  $x$ -axis and the parabola opens downward

**Section 9.2    Page 486    Question 15**

Answers may vary. Examples:

**a)** Build an inequality that has solution  $-2 \leq x \leq 7$ .

The interval between the roots of a positive parabola with roots  $-2$  and  $7$  will work.

$$(x + 2)(x - 7) \leq 0$$

$$x^2 - 5x - 14 \leq 0$$

**b)** Build an inequality that has solution  $x < 1$  or  $x > 10$ .

The intervals outside the roots of a positive parabola with roots  $1$  and  $10$  will work.

$$(x - 1)(x - 10) > 0$$

$$x^2 - 11x + 10 > 0$$

**c)** Build an inequality that has solution  $\frac{5}{3} \leq x \leq 6$ .

The interval between the roots of a positive parabola with roots  $\frac{5}{3}$  and  $6$  will work.

$$(3x - 5)(x - 6) \leq 0$$

$$3x^2 - 23x + 30 \leq 0$$

**d)** Build an inequality that has solution  $x < -\frac{3}{4}$  or  $x > -\frac{1}{5}$ .

The intervals outside the roots of a positive parabola with roots  $-\frac{3}{4}$  and  $-\frac{1}{5}$  will work

$$(4x + 3)(5x + 1) > 0$$

$$20x^2 + 19x + 3 > 0$$

**e)** Build an inequality that has solution  $x \leq -3 - \sqrt{7}$  or  $x \geq -3 + \sqrt{7}$ .

The intervals outside the roots of a positive parabola with roots  $-3 - \sqrt{7}$  and  $-3 + \sqrt{7}$  will work.

$$(x + 3 + \sqrt{7})(x + 3 - \sqrt{7}) \geq 0$$

$$x^2 + 6x + 2 \geq 0$$

**f)** Build an inequality that has solution  $x \in \mathbb{R}$ .

Any positive parabola with vertex above the  $x$ -axis will work.

$$(x - 2)^2 + 3 > 0$$

**g)** Build an inequality that has no solution. The equation from part f) is entirely above the  $x$ -axis so it is never negative.

$$(x - 2)^2 + 3 \leq 0 \text{ has no solution.}$$

**Section 9.2 Page 487 Question 16**

$$|x^2 - 4| \geq 2$$

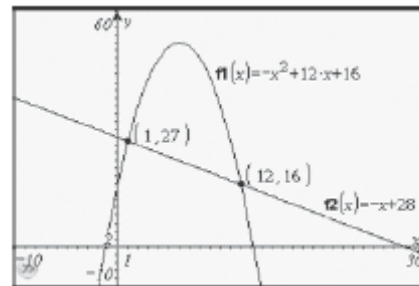
$$\text{Either } x^2 - 4 \geq 2 \quad \text{or} \quad x^2 - 4 \leq -2$$

$$x^2 \geq 6 \quad \text{or} \quad x^2 \leq 2$$

$$x \leq -\sqrt{6} \text{ or } x \geq \sqrt{6} \quad \text{or} \quad -\sqrt{2} \leq x \leq \sqrt{2}$$

**Section 9.2 Page 487 Question 17**

**a)** For  $-x^2 + 12x + 16 \geq -x + 28$ , the solution is  $1 \leq x \leq 12$  because the graph shows that in that interval the parabola is above and hence greater than the line.



**b)**  $-x^2 + 12x + 16 \geq -x + 28$   
 $-x^2 + 13x - 12 \geq 0$   
or  $x^2 - 13x + 12 \leq 0$

**c)**  $(x - 12)(x - 1) \leq 0$

*Case 1:* The first factor is negative and second factor is positive.

$$x - 12 \leq 0 \text{ and } x - 1 \geq 0$$

$$x \leq 12 \text{ and } x \geq 1$$

These two conditions are true when  $1 \leq x \leq 12$ .

*Case 2:* The first factor is positive and second factor is negative.

$$x - 12 \geq 0 \text{ and } x - 1 \leq 0$$

$$x \geq 12 \text{ and } x \leq 1$$

These conditions are never both true.

The solution set is  $\{x \mid 1 \leq x \leq 12, x \in \mathbb{R}\}$ .

**d)** The solutions in parts a) and c) are the same. The second inequality is derived from the first and is equivalent to it.

**Section 9.2 Page 487 Question 18**

In all of the solution methods the first step is to rearrange the terms so they are all on one side of the inequality sign. You need to be able to identify the related quadratic function  $f(x) = 2x^2 - 7x - 12$  for the graphing method. For either roots and test points, sign, or case analysis you need to factor the related quadratic.

**Section 9.2 Page 487 Question 19**

Answers may vary. Example: Graphing works well if you have a graphing calculator available. When the related quadratic can easily be factored then case analysis is quite straightforward and is the method I prefer. Using roots and test points is quite time consuming and sign analysis requires more thoughtful logical steps.

**Section 9.2 Page 487 Question 20**

**a)** Devan made a mistake in the first step; he reversed the inequality sign when he should not have done. The inequality should be  $x^2 + 5x + 6 \leq 0$ .

The solution continues:

$$(x + 2)(x + 3) \leq 0$$

*Case 1:* The first factor is negative and second factor is positive.

$$x + 2 \leq 0 \text{ and } x + 3 \geq 0$$

$$x \leq -2 \text{ and } x \geq -3$$

These conditions are both true when  $-3 \leq x \leq -2$ .

*Case 2:* The first factor is positive and second factor is negative.

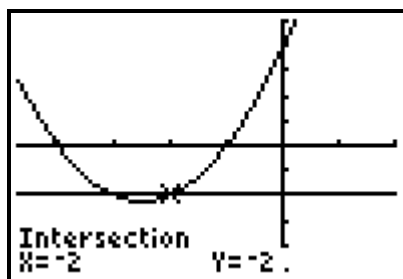
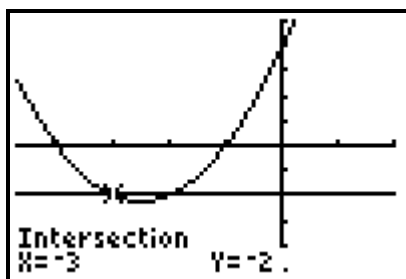
$$x + 2 \geq 0 \text{ and } x + 3 \leq 0$$

$$x \geq -2 \text{ and } x \leq -3$$

These conditions are never both true.

So, the solution set is  $\{x \mid -3 \leq x \leq -2, x \in \mathbb{R}\}$ .

**b)** Graph  $f(x) = x^2 + 5x + 4$  and  $g(x) = -2$ . Identify the interval where the parabola is below the line as that is where  $x^2 + 5x + 4 \leq -2$ .



The parabola is below the line  $y = -2$ , when  $-3 \leq x \leq -2$ . This confirms the answer from part a).

## Section 9.3 Quadratic Inequalities in Two Variables

### Section 9.3 Page 496 Question 1

**a)**  $y < x^2 + 3$

Test (2, 6).

Left Side	Right Side
y	$x^2 + 3$
= 6	= $(2)^2 + 3$
	= 7

Left Side < Right Side

Test (-1, 3).

Left Side	Right Side
y	$x^2 + 3$
= 3	= $(-1)^2 + 3$
	= 4

Left Side < Right Side

The points (2, 6) and (-1, 3) are solutions to the inequality  $y < x^2 + 3$ .

Test (4, 20).

Left Side	Right Side
y	$x^2 + 3$
= 20	= $(4)^2 + 3$
	= 19

Left Side > Right Side

Test (-3, 12).

Left Side	Right Side
y	$x^2 + 3$
= 12	= $(-3)^2 + 3$
	= 12

Left Side = Right Side

**b)**  $y \leq -x^2 + 3x - 4$

Test (2, -2).

Left Side	Right Side
y	$-x^2 + 3x - 4$
= -2	= $-(2)^2 + 3(2) - 4$
	= -6

Left Side < Right Side

Test (4, -1).

Left Side	Right Side
y	$-x^2 + 3x - 4$
= -1	= $-(4)^2 + 3(4) - 4$
	= -8

Left Side > Right Side

Test (0, -6).

Left Side	Right Side
y	$-x^2 + 3x - 4$
= -6	= $-(0)^2 + 3(0) - 4$
	= -4

Left Side < Right Side

Test (-2, -15).

Left Side	Right Side
y	$-x^2 + 3x - 4$
= -15	= $-(-2)^2 + 3(-2) - 4$
	= -14

Left Side < Right Side

The points (2, -2), (0, -6), and (-2, -15) are solutions to the inequality  $y \leq -x^2 + 3x - 4$ .

**c)**  $y > 2x^2 + 3x + 6$

Test (-3, 5).

Left Side	Right Side
y	$2x^2 + 3x + 6$
= 5	= $2(-3)^2 + 3(-3) + 6$

Test (0, -6).

Left Side	Right Side
y	$2x^2 + 3x + 6$
= -6	= $2(0)^2 + 3(0) + 6$

$$= 15$$

Left Side < Right Side

Test (2, 10).

Left Side	Right Side
y	$2x^2 + 3x + 6$
= 10	$= 2(\textcolor{red}{2})^2 + 3(\textcolor{red}{2}) + 6$
	= 20

Left Side < Right Side

None of the points suggested is a solution to the inequality  $y > 2x^2 + 3x + 6$ .

$$= 6$$

Left Side < Right Side

Test (4, 40).

Left Side	Right Side
y	$2x^2 + 3x + 6$
= 40	$= 2(\textcolor{red}{4})^2 + 3(\textcolor{red}{4}) + 6$
	= 50

Left Side < Right Side

**d)**  $y \geq -\frac{1}{2}x^2 - x + 5$

Test (-4, 2).

Left Side	Right Side
y	$-0.5x^2 - x + 5$
= 2	$= -0.5(\textcolor{red}{-4})^2 - (\textcolor{red}{-4}) + 5$
	= 1

Left Side > Right Side

Test (-1, 5).

Left Side	Right Side
y	$-0.5x^2 - x + 5$
= 5	$= -0.5(\textcolor{red}{-1})^2 - (\textcolor{red}{-1}) + 5$
	= 6.5

Left Side < Right Side

Test (1, 3.5).

Left Side	Right Side
y	$-0.5x^2 - x + 5$
= 3.5	$= -0.5(\textcolor{red}{1})^2 - (\textcolor{red}{1}) + 5$
	= 3.5

Left Side = Right Side

Test (3, 2.5).

Left Side	Right Side
y	$-0.5x^2 - x + 5$
= 2.5	$= -0.5(\textcolor{red}{3})^2 - (\textcolor{red}{3}) + 5$
	= -2.5

Left Side > Right Side

The points (-4, 2), (1, 3.5) and (3, 2.5) are solutions to the inequality  $y \geq -\frac{1}{2}x^2 - x + 5$ .

### Section 9.3 Page 496 Question 2

**a)**  $y \geq 2(x - 1)^2 + 1$

Test (0, 1).

Left Side	Right Side
y	$2(x - 1)^2 + 1$
= 1	$= 2(\textcolor{red}{0} - 1)^2 + 1$
	= 3

Left Side < Right Side

Test (1, 0).

Left Side	Right Side
y	$2(x - 1)^2 + 1$
= 0	$= 2(\textcolor{red}{1} - 1)^2 + 1$
	= 1

Left Side < Right Side

Test (3, 6).

Left Side	Right Side
y	$2(x - 1)^2 + 1$

Test (-2, 15).

Left Side	Right Side
y	$2(x - 1)^2 + 1$

$$= 6 \qquad = 2(3 - 1)^2 + 1 \\ = 9$$

Left Side < Right Side

None of the points (0, 1), (1, 0), (3, 6) and (-2, 15) is a solution to the inequality  $y \geq 2(x - 1)^2 + 1$ .

$$= 15 \qquad = 2(-2 - 1)^2 + 1 \\ = 19$$

Left Side < Right Side

**b)**  $y > -(x + 2)^2 - 3$

Test (-3, 1).

Left Side	Right Side
y	$-(x + 2)^2 - 3$
= 1	$= 2(-3 + 2)^2 - 3$
	= -1

Left Side > Right Side

Test (-2, -3).

Left Side	Right Side
y	$-(x + 2)^2 - 3$
= -3	$= -(-2 + 2)^2 - 3$
	= -3

Left Side = Right Side

Test (0, -8).

Left Side	Right Side
y	$-(x + 2)^2 - 3$
= -8	$= 2(0 + 2)^2 - 3$
	= 5

Left Side < Right Side

Test (1, 2).

Left Side	Right Side
y	$-(x + 2)^2 - 3$
= 2	$= -(1 + 2)^2 - 3$
	= -12

Left Side > Right Side

The points (-2, -3) and (0, -8) are not solutions to the inequality  $y > -(x + 2)^2 - 3$ .

**c)**  $y \leq \frac{1}{2}(x - 4)^2 + 5$

Test (0, 4).

Left Side	Right Side
y	$\frac{1}{2}(x - 4)^2 + 5$
= 4	$= 0.5(0 - 4)^2 + 5$
	= 13

Left Side < Right Side

Test (3, 1).

Left Side	Right Side
y	$\frac{1}{2}(x - 4)^2 + 5$
= 1	$= 0.5(3 - 4)^2 + 5$
	= 5.5

Left Side < Right Side

Test (4, 5).

Left Side	Right Side
y	$\frac{1}{2}(x - 4)^2 + 5$
= 5	$= 0.5(4 - 4)^2 + 5$
	= 5

Left Side = Right Side

Test (2, 9).

Left Side	Right Side
y	$\frac{1}{2}(x - 4)^2 + 5$
= 9	$= 0.5(2 - 4)^2 - 3$
	= -5

Left Side > Right Side

The ordered pair (2, 9) is not a solution to the inequality  $y \leq \frac{1}{2}(x - 4)^2 + 5$ .

**d)**  $y < -\frac{2}{3}(x+3)^2 - 2$

Test  $(-2, 2)$ .

Left Side	Right Side
$y$	$-\frac{2}{3}(x+3)^2 - 2$
$= 2$	$= -\frac{2}{3}(-2+3)^2 - 2$
	$= -2\frac{2}{3}$

Left Side  $>$  Right Side

Test  $(-1, -5)$ .

Left Side	Right Side
$y$	$-\frac{2}{3}(x+3)^2 - 2$
$= -5$	$= -\frac{2}{3}(-1+3)^2 - 2$
	$= -4\frac{2}{3}$

Left Side  $<$  Right Side

Test  $(-3, -2)$ .

Left Side	Right Side
$y$	$-\frac{2}{3}(x+3)^2 - 2$
$= -2$	$= -\frac{2}{3}(-3+3)^2 - 2$
	$= -2$

Left Side = Right Side

Test  $(0, -10)$ .

Left Side	Right Side
$y$	$-\frac{2}{3}(x+3)^2 - 2$
$= -10$	$= -\frac{2}{3}(0+3)^2 - 2$
	$= -8$

Left Side  $<$  Right Side

The ordered pairs  $(-2, 2)$  and  $(-3, -2)$  are not solutions to the inequality

$$y < -\frac{2}{3}(x+3)^2 - 2.$$

### Section 9.3 Page 497 Question 3

**a)** The shaded region is below the dashed line boundary, so it is described by the inequality  $y < -x^2 - 4x + 5$ .

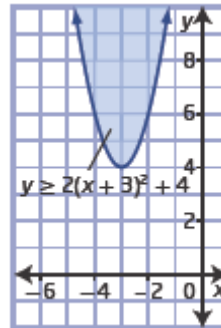
**b)** The shaded region is below the solid line boundary, so it is described by the inequality  $y \leq \frac{1}{2}x^2 - x + 3$ .

**c)** The shaded region is above the solid line boundary, so it is described by the inequality  $y \geq -\frac{1}{4}x^2 - x + 3$ .

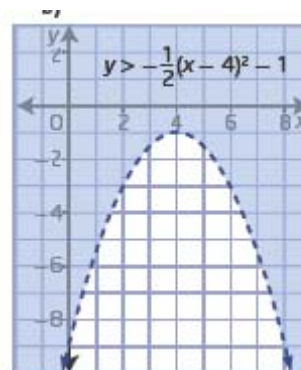
**d)** The shaded region is above the dashed line boundary, so it is described by the inequality  $y > 4x^2 + 5x - 6$ .

**Section 9.3 Page 497 Question 4**

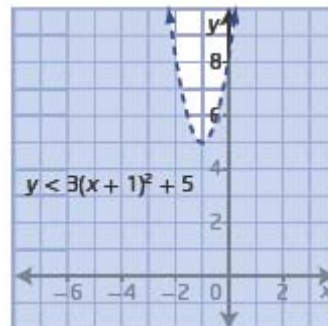
**a)** From the boundary equation, the vertex is at  $(-3, 4)$  and the parabola opens upward. It is twice as steep as  $y = x^2$ . Since the inequality has “greater than or equals” use a solid boundary line and shade above the line.



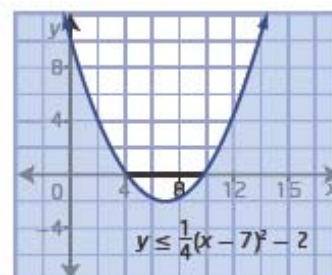
**b)** From the boundary equation, the vertex is at  $(4, -1)$  and the parabola opens downward. It is half as steep as  $y = x^2$ . Since the inequality has “greater than” use a dashed boundary line and shade above the line.



**c)** From the boundary equation, the vertex is at  $(-1, 5)$  and the parabola opens upward. It is 3 times as steep as  $y = x^2$ . Since the inequality has “less than” use a dashed boundary line and shade below the line.



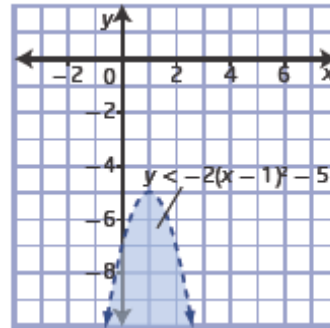
**d)** From the boundary equation, the vertex is at  $(7, -2)$  and the parabola opens upward. It is one-quarter as steep as  $y = x^2$ . Since the inequality has “less than or equals” use a solid boundary line and shade below the line.



**Section 9.3 Page 497 Question 5**

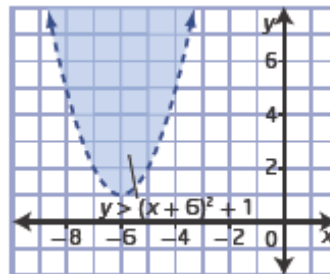
**a)**  $y < -2(x - 1)^2 - 5$

For the boundary, the vertex is at  $(1, -5)$  and the parabola opens downward. When  $x = 0$ ,  $y = -7$ . By symmetry another point is  $(2, -7)$ . Use a dashed boundary line and shade below the parabola to show  $y < -2(x - 1)^2 - 5$ .



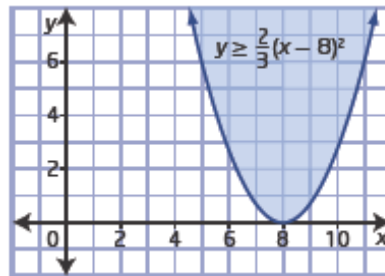
**b)**  $y > (x + 6)^2 + 1$

For the boundary, the vertex is at  $(-6, 1)$  and the parabola opens upward. When  $x = -5$ ,  $y = 2$ . By symmetry another point is  $(-7, 2)$ . Use a dashed boundary line and shade above the parabola to show  $y > (x + 6)^2 + 1$ .



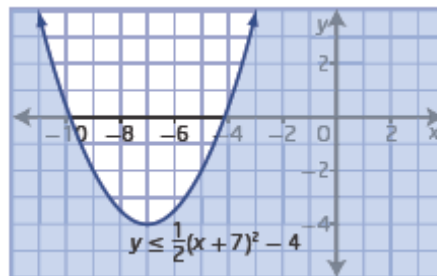
**c)**  $y \geq \frac{2}{3}(x - 8)^2$

For the boundary, the vertex is at  $(8, 0)$  and the parabola opens upward. When  $x = 11$ ,  $y = 6$ . By symmetry another point is  $(5, 6)$ . Use a solid boundary line and shade above the parabola to show  $y \geq \frac{2}{3}(x - 8)^2$ .



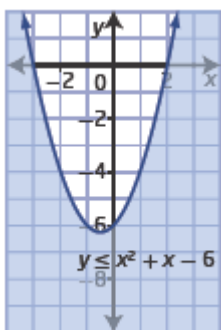
**d)**  $y \leq \frac{1}{2}(x + 7)^2 - 4$

For the boundary, the vertex is at  $(-7, -4)$  and the parabola opens upward. When  $x = -5$ ,  $y = -2$ . By symmetry another point is  $(-9, -2)$ . Use a solid boundary line and shade below the parabola to show  $y \leq \frac{1}{2}(x + 7)^2 - 4$ .

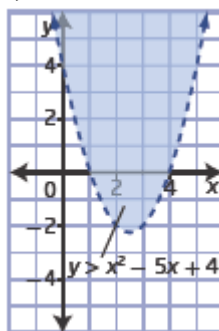


Section 9.3 Page 497 Question 6

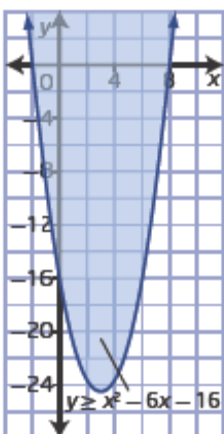
a)



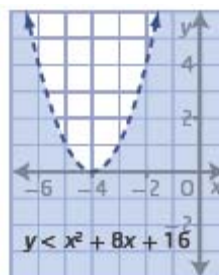
b)



c)

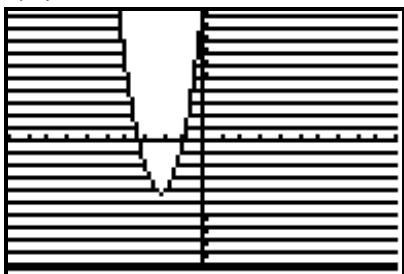


d)



Section 9.3 Page 497 Question 7

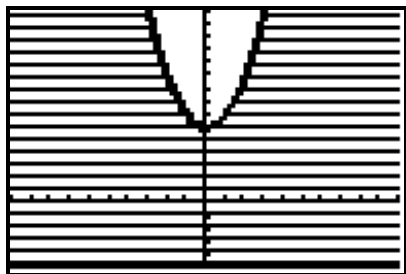
a)  $y < 3x^2 + 13x + 10$



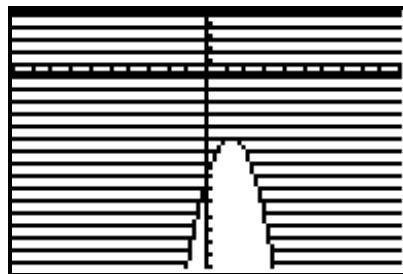
b)  $y \geq -x^2 + 4x + 7$



c)  $y \leq x^2 + 6$



d)  $y > -2x^2 + 5x - 8$



**Section 9.3 Page 497 Question 8**

a) The vertex of the boundary is at  $(0, 1)$ , so the quadratic has the form  $y = ax^2 + 1$ . The point  $(2, 3)$  is on the parabola so substitute  $x = 2$  and  $y = 3$ .

$$3 = a(2)^2 + 1$$

$$2 = 4a$$

$$a = \frac{1}{2}$$

The boundary is a solid line and the region above it is shaded. So, the inequality that describes the graph is  $y \geq \frac{1}{2}x^2 + 1$ .

b) The  $x$ -intercepts of the boundary quadratic are  $-2$  and  $4$ , so its equation has the form  $y = a(x + 2)(x - 4)$ . The vertex of the boundary is at  $(1, 3)$ , so substitute  $x = 1$  and  $y = 3$ .

$$3 = a(1 + 2)(1 - 4)$$

$$3 = -9a$$

$$a = -\frac{1}{3}$$

The boundary is a broken line and the region above it is shaded. So, the inequality that describes the graph is  $y > -\frac{1}{3}(x + 2)(x - 4)$  or  $y > -\frac{1}{3}(x^2 - 2x - 8)$ .

**Section 9.3 Page 498 Question 9**

a) The vertex of the parabola is at  $(50, 4)$ , so the function has the form  $y = a(x - 50)^2 + 4$ . The point  $(0, 0)$  is on the curve, so substitute  $x = 0$  and  $y = 0$ .

$$0 = a(0 - 50)^2 + 4$$

$$-4 = 2500a$$

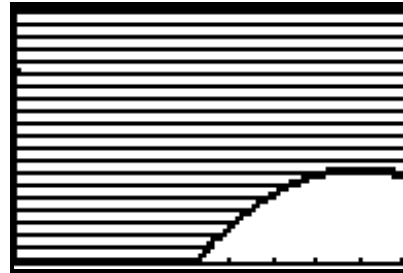
$$a = -\frac{1}{625}$$

The function that models the parabolic arch is  $y = -\frac{1}{625}(x-50)^2 + 4$ .

- b) The region below the arch can be modelled by the inequality  $y < -\frac{1}{625}(x-50)^2 + 4$ ,  $0 \leq x \leq 100$ .

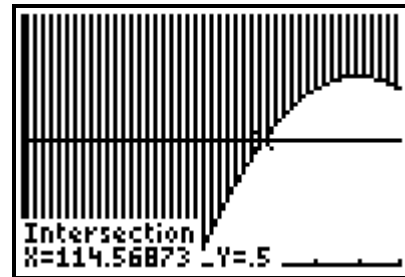
**Section 9.3 Page 498 Question 10**

- a)  $L \geq -0.000125a^2 + 0.040a - 2.442$ ,  
where  $L \geq 0$  because  $L$  is an area and  
 $0 \leq a \leq 180$  because  $a$  represents the hip  
angle.



- b) Enter the second function  $y = 0.5$  and  
determine what values of  $a$  are above this  
line.

Any hip angle of about  $114.6^\circ$  up to  $180^\circ$   
will generate a lift area of at least  $0.50 \text{ m}^2$ .



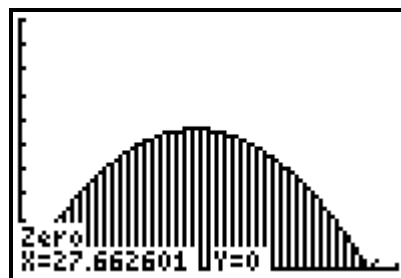
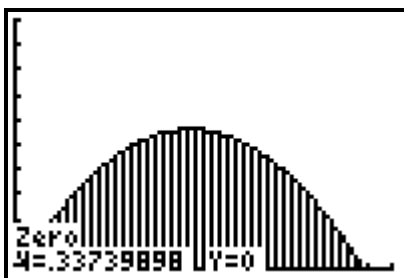
**Section 9.3 Page 498 Question 11**

- a) The possible water levels,  $y$ , below the arch of the bridge are given by  
 $y < -0.03x^2 + 0.84x - 0.08$

- b) Substitute  $y = 0.2$ .

$$0.2 \leq -0.03x^2 + 0.84x - 0.08$$

$$0 \leq -0.03x^2 + 0.84x - 0.28$$



For  $0.337 \leq x \leq 27.663$ , the river has its normal level of  $0.2 \text{ m}$ .

c) The width of the river, under the arch, in the situation described in part b) is approximately  $27.663 - 0.337$  or  $27.326$  m.

**Section 9.3 Page 499 Question 12**

a)  $h = -2.944t^2 + 191.360t + 6950.400$

If they experience weightlessness when the jet is above 9600 m, then

$$9600 < -2.944t^2 + 191.360t + 6950.400$$

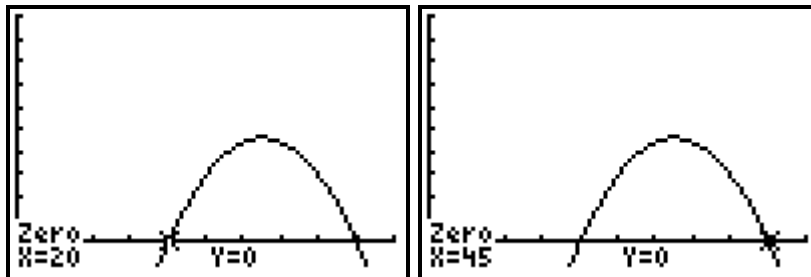
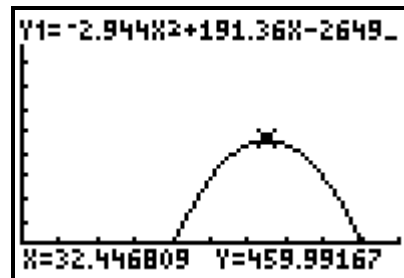
$$0 < -2.944t^2 + 191.360t - 2649.6$$

b)

Graph the boundary

$$y = -2.944t^2 + 191.360t - 2649.6$$

and determine when the function is above the  $x$ -axis. The jet is at or above 9600 m between 20 s and 45 s.



c) Microgravity exists for  $45 \text{ s} - 20 \text{ s}$ , or  $25 \text{ s}$ .

**Section 9.3 Page 499 Question 13**

a) The equation for the arch has the form

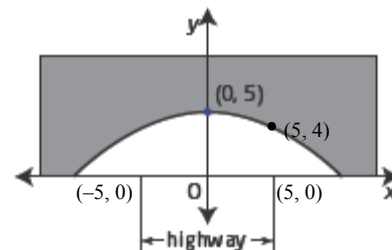
$$y = ax^2 + 5.$$

Since the road is 10 m wide and the minimum height over the road is 4 m,  $(5, 4)$  is a point on the parabolic arch. Substitute to determine  $a$ .

$$4 = a(5)^2 + 5$$

$$a = -\frac{1}{25} \text{ or } -0.04$$

The equation for the parabolic arch is  $y = -0.04x^2 + 5$ .



b) The inequality that represent the space under the bridge and in quadrants I and II is

$$0 \leq -0.04x^2 + 5.$$

a) Use the three given points to determine the quadratic equation. Let  $x$  represent the number of ads and  $y$  the revenue. One  $x$ -intercept is at 0, so the equation has the form  $y = ax(x - b)$ , where  $b$  is the other  $x$ -intercept.

To determine  $a$  and  $b$  substitute the other two points (10, 100) and (15, 75).

$$100 = 10a(10 - b) \quad 75 = 15a(15 - b)$$

$$10 = 10a - ab \quad ① \quad 5 = 15a - ab \quad ②$$

Subtract equations ② - ①.

$$-5 = 5a$$

$$a = -1$$

Substitute back in ① to determine  $b$ .

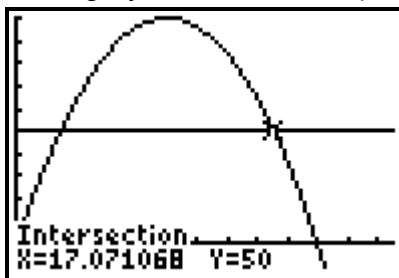
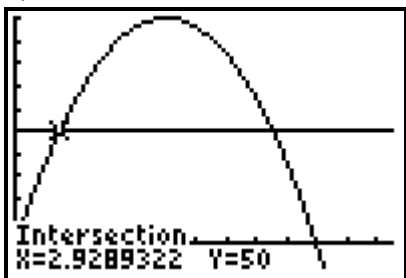
$$10 = 10(-1) - (-1)b$$

$$20 = b$$

So, the equation for the quadratic path of the data is  $y = -x(x - 20)$  or  $y = -x^2 + 20x$ .

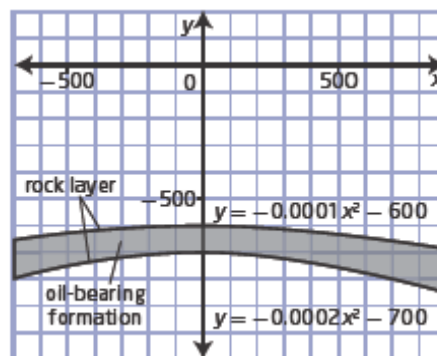
The quadratic inequality for Tavia's revenue based on the data so far is  $y \leq -x^2 + 20x$ .

b) Determine what values of  $x$  give  $y \geq 50$ . Graph  $y = -x^2 + 20x$  and  $y = 50$ .



Since  $x$  represents the number of ads it must be a whole number. So for 3 ads up to and including 17 ads, the revenue is at least \$50.

The oil bearing formation is the region between the two quadratic boundaries and so is described by  $y \leq -0.0001x^2 - 600$  and  $y \geq -0.0002x^2 - 700$ , or  $-0.0002x^2 - 700 \leq y \leq -0.0001x^2 - 600$ .



**Section 9.3   Page 500   Question 16**

a) Let  $x$  represent the number of \$0.50 increases and  $y$  the revenue from candy-gram sales.

Revenue = (number sold)(price each)

$$y = (400 - 20x)(4 + 0.5x)$$

$$y = -10x^2 + 120x + 1600$$

b) If the student council wants revenue of at least \$1800, then

$$-10x^2 + 120x + 1600 \geq 1800$$

$$-10x^2 + 120x - 200 \geq 0$$

$$x^2 - 12x + 20 \leq 0$$

$$(x - 10)(x - 2) \leq 0$$

Case 1: first factor is positive and second factor is negative

$$x - 10 \geq 0 \text{ and } x - 2 \leq 0$$

$$x \geq 10 \text{ and } x \leq 2$$

These two inequalities are never both true.

Case 2: first factor is negative and second factor is positive

$$x - 10 \leq 0 \text{ and } x - 2 \geq 0$$

$$x \leq 10 \text{ and } x \geq 2$$

These two conditions are true when  $2 \leq x \leq 10$ .

Since  $x$  represents an increase of \$0.50, prices of between  $\$4 + 2(0.50)$  and  $\$4 + 10(0.50)$  will give at least \$1800. Prices between \$5 and \$9 will give at least \$1800.

c) If the student council wants revenue of at least \$1600, then

$$-10x^2 + 120x + 1600 \geq 1600$$

$$-10x^2 + 120x \geq 0$$

$$x^2 - 12x \leq 0$$

$$x(x - 12) \leq 0$$

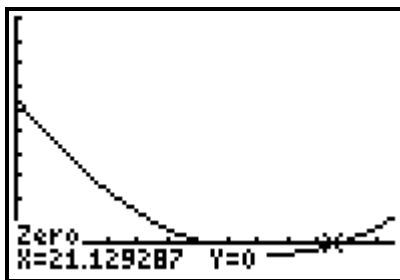
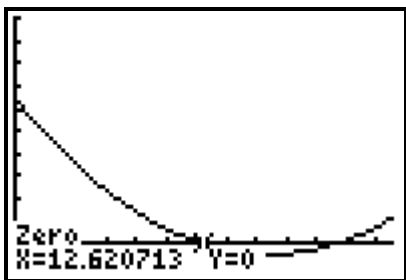
Similar case analysis as in part b), gives the inequality is true when  $0 \leq x \leq 12$ . This means that prices of between \$4 and \$10 will raise \$1600 or more.

**Section 9.3   Page 500   Question 17**

a)  $p \leq 0.24t^2 - 8.1t + 74$

Determine when  $0.24t^2 - 8.1t + 74 \leq 10$

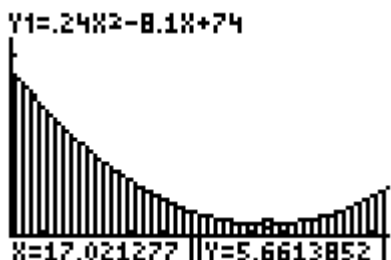
Use a graph to determine when  $0.24t^2 - 8.1t + 64 \leq 0$ .



Production is below 10% of peak production from about 12.6 years until 21.1 years after 2000.

b) The environmentalist used the inequality  $p \leq 0.24t^2 - 8.1t + 74$ .

The percent of methane produced keeps decreasing for about 17 years, from 74% in 2000 to about 5.7% in 2017. This is the part of the graph that provides a reasonable model of the situation.



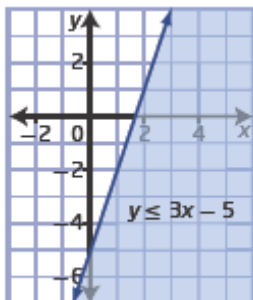
c) Based on part b), the end date in part a) should be modified. We can probably say that production is below 10% of peak production from about 12.6 years until 17.0 years after 2000.

d) He could restrict the domain to values of  $t$ , or times, between 0 and 17 years.

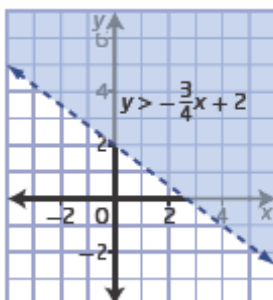
## Chapter 9 Review

### Chapter 9 Review Page 501 Question 1

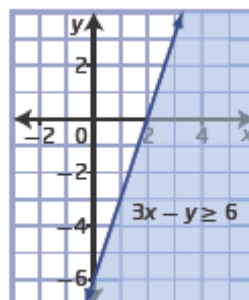
a)



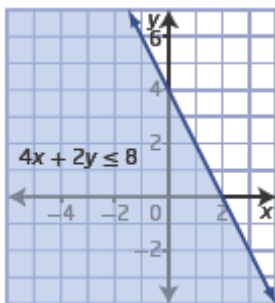
b)



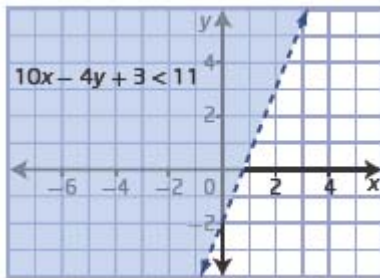
c)



d)

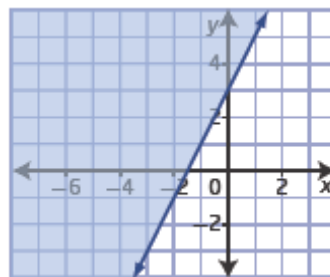


e)

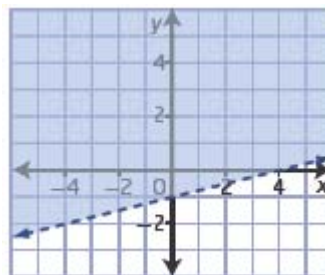


**Chapter 9 Review Page 501 Question 2**

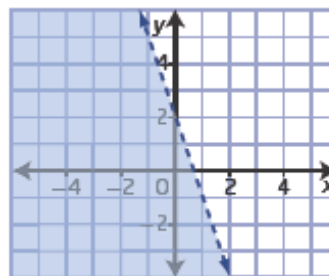
a) The boundary line has y-intercept 3 and slope 2. The boundary equation is  $y = 2x + 3$ . Since the region shade is above a solid line, its inequality is  $y \geq 2x + 3$ .



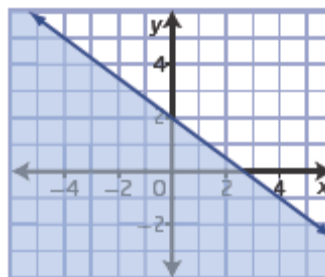
b) The boundary line has y-intercept  $-1$  and slope  $\frac{1}{4}$ . The boundary equation is  $y = \frac{1}{4}x - 1$ . Since the region shade is above a broken line, its inequality is  $y > \frac{1}{4}x - 1$ .



c) The boundary line has y-intercept 2 and slope  $-3$ . The boundary equation is  $y = -3x + 2$ . Since the region shade is below a broken line, its inequality is  $y < -3x + 2$ .



**d)** The boundary line has y-intercept 2 and slope  $-\frac{3}{4}$ . The boundary equation is  $y = -\frac{3}{4}x + 2$ . Since the region shade is below a solid line, its inequality is  $y \leq -\frac{3}{4}x + 2$ .

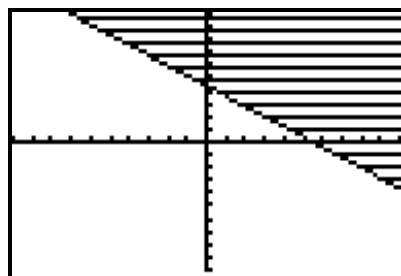


**Chapter 9 Review Page 501 Question 3**

**a)**  $4x + 5y > 22$

$$5y > -4x + 22$$

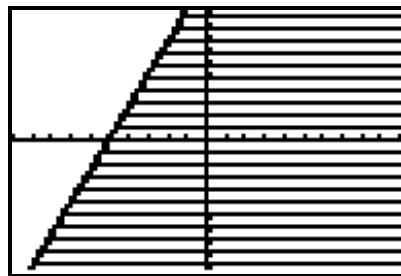
$$y > -\frac{4}{5}x + \frac{22}{5}$$



**b)**  $10x - 4y + 52 \geq 0$

$$10x + 52 \geq 4y$$

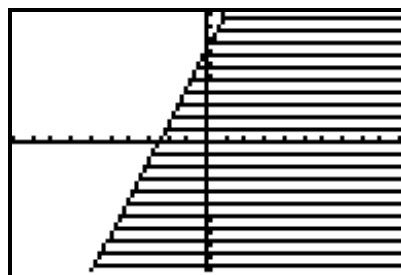
$$y \leq \frac{5}{2}x + 13$$



**c)**  $-3.2x + 1.1y < 8$

$$11y < 32x + 80$$

$$y < \frac{32}{11}x + \frac{80}{11}$$

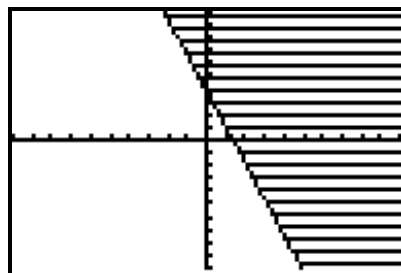


**d)**  $12.4x + 4.4y > 16.5$

$$44y > -124x + 165$$

$$y > -\frac{124}{44}x + \frac{165}{44}$$

$$y > -\frac{31}{11}x + \frac{15}{4}$$



e)  $\frac{3}{4}x \leq 9y$   
 $y \geq \frac{1}{12}x$



**Chapter 9 Review Page 501 Question 4**

- a) Let  $x$  represent the number of movies and  $y$  the number of meals. Then, with a budget of \$120,  
 $15x + 10y \leq 120$



- c) Any whole number values in quadrant I, on or below the boundary line give the numbers of movies and meals that Janelle can afford on her budget.

**Chapter 9 Review Page 501 Question 5**

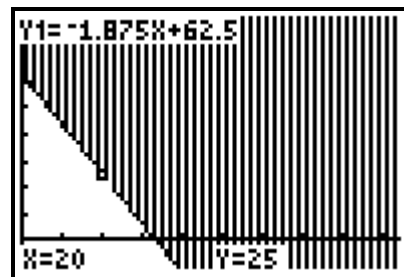
- a) Commission on each laptop =  $0.05(600)$   
 $= 30$

Commission on each DVD player =  $0.08(200)$   
 $= 16$

- b) Let  $x$  represent the number of laptops that Jodi sells and  $y$  the number of DVD players that she sells. If she wants to earn a minimum commission of \$1000, then  
 $30x + 16y \geq 1000$ .

- c)  $16y \geq -30x + 1000$   
 $y \geq -1.875x + 62.5$

For any combination of whole number values that fall in quadrant I and on or above the line, Jodi will earn \$1000 commission or more.



Methods may vary.

**a)**  $x^2 - 2x - 63 > 0$

Solve by factoring and considering cases, because the quadratic is easy to factor.

$$(x - 9)(x + 7) > 0$$

*Case 1:* Both factors are positive.

$$x - 9 > 0 \text{ and } x + 7 > 0$$

$$x > 9 \text{ and } x > -7$$

Both conditions are true when  $x > 9$ .

*Case 2:* Both factors are negative.

$$x - 9 < 0 \text{ and } x + 7 < 0$$

$$x < 9 \text{ and } x < -7$$

Both conditions are true when  $x < -7$ .

So the solution set for  $x^2 - 2x - 63 > 0$  is  $\{x \mid x < -7 \text{ or } x > 9, x \in \mathbb{R}\}$ .

**b)**  $2x^2 - 7x - 30 \geq 0$

Solve by factoring and considering cases, because the quadratic can be factored.

$$(2x + 5)(x - 6) \geq 0$$

*Case 1:* Both factors are positive.

$$2x + 5 \geq 0 \text{ and } x - 6 \geq 0$$

$$x \geq -2.5 \text{ and } x \geq 6$$

Both conditions are true when  $x \geq 6$ .

*Case 2:* Both factors are negative.

$$2x + 5 \leq 0 \text{ and } x - 6 \leq 0$$

$$x \leq -2.5 \text{ and } x \leq 6$$

Both conditions are true when  $x \leq -2.5$ .

So the solution set for  $2x^2 - 7x - 30 \geq 0$  is  $\{x \mid x \leq -2.5 \text{ or } x \geq 6, x \in \mathbb{R}\}$ .

**c)**  $x^2 + 8x - 48 < 0$

Solve by factoring and considering cases, because the quadratic is easy to factor.

$$(x + 12)(x - 4) < 0$$

*Case 1:* First factor is positive and second factor is negative.

$$x + 12 > 0 \text{ and } x - 4 < 0$$

$$x > -12 \text{ and } x < 4$$

Both conditions are true when  $-12 < x < 4$ .

*Case 2:* First factor is negative and second factor is positive.

$$x + 12 < 0 \text{ and } x - 4 > 0$$

$$x < -12 \text{ and } x > 4$$

These two inequalities are never both true.

So the solution set for  $x^2 + 8x - 48 < 0$  is  $\{x \mid -12 < x < 4, x \in \mathbb{R}\}$ .

**d)**  $x^2 - 6x + 4 \geq 0$

The quadratic does not factor, so use the quadratic formula to find the roots and then test points either side of the roots.

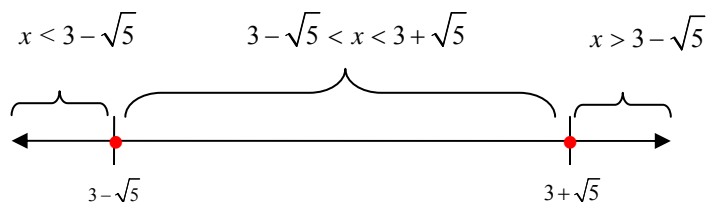
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(4)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{20}}{2}$$

$$x = 3 \pm \sqrt{5}$$

The roots are approximately 0.76 and 5.24.



Choose test points in each of the three intervals.

Interval	$x < 3 - \sqrt{5}$	$3 - \sqrt{5} < x < 3 + \sqrt{5}$	$x > 3 + \sqrt{5}$
Test Point	0	1	6
Substitution	$x^2 - 6x + 4$ $= (-0)^2 - 6(0) + 4$ $= 4$	$x^2 - 6x + 4$ $= (1)^2 - 6(1) + 4$ $= -1$	$x^2 - 6x + 4$ $= (6)^2 - 6(6) + 4$ $= 4$
Is $x^2 - 6x + 4 \geq 0$ ?	yes	no	yes

The solution set for  $x^2 + 8x - 48 < 0$  is  $\{x \mid x \leq 3 - \sqrt{5} \text{ or } x \geq 3 + \sqrt{5}, x \in \mathbb{R}\}$ .

## Chapter 9 Review Page 502 Question 7

**a)**  $x(6x + 5) \leq 4$

$$6x^2 + 5x - 4 \leq 0$$

$$(3x + 4)(2x - 1) \leq 0$$

Case 1: The first factor is positive and the second factor is negative.

$$3x + 4 \geq 0 \text{ and } 2x - 1 \leq 0$$

$$x \geq -\frac{4}{3} \text{ and } x \leq \frac{1}{2}$$

These conditions are both true when  $-\frac{4}{3} \leq x \leq \frac{1}{2}$ .

Case 2: The first factor is negative and the second factor is positive.

$$3x + 4 \leq 0 \text{ and } 2x - 1 \geq 0$$

$$x \leq -\frac{4}{3} \text{ and } x \geq \frac{1}{2}$$

These two conditions are never both true.

So the solution set for  $x(6x + 5) \leq 4$  is  $\{x \mid -\frac{4}{3} \leq x \leq \frac{1}{2}, x \in \mathbb{R}\}$ .

**b)**  $4x^2 < 10x - 1$

$$4x^2 - 10x + 1 < 0$$

Use the quadratic formula to find the roots.

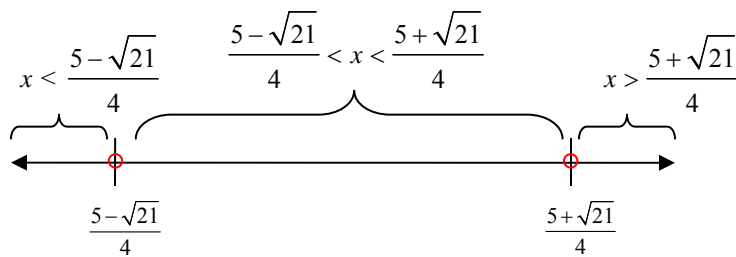
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(4)(1)}}{2(4)}$$

$$x = \frac{10 \pm \sqrt{84}}{8}$$

$$x = \frac{5 \pm \sqrt{21}}{4}$$

The roots are approximately 0.10 and 5.73.



Choose test points in each of the three intervals.

Interval	$x < \frac{5 - \sqrt{21}}{4}$	$\frac{5 - \sqrt{21}}{4} < x < \frac{5 + \sqrt{21}}{4}$	$x > \frac{5 + \sqrt{21}}{4}$
Test Point	0	1	6
Substitution	$4x^2 = 4(0)^2 = 0$ $10x - 1$ $= 10(0) - 1$ $= -1$	$4x^2 = 4(1)^2 = 4$ $10x - 1$ $= 10(1) - 1$ $= 9$	$4x^2 = 4(6)^2 = 144$ $10x - 1$ $= 10(6) - 1$ $= 59$
Is $4x^2 < 10x - 1$ ?	no	yes	no

The solution set for  $4x^2 < 10x - 1$  is  $\{x \mid \frac{5-\sqrt{21}}{4} < x < \frac{5+\sqrt{21}}{4}, x \in \mathbb{R}\}$ .

**c)**  $x^2 \leq 4(x + 8)$

$$x^2 - 4x - 32 \leq 0$$

$$(x - 8)(x + 4) \leq 0$$

*Case 1:* The first factor is positive and second factor is negative.

$$x - 8 \geq 0 \text{ and } x + 4 \leq 0$$

$$x \geq 8 \text{ and } x \leq -4$$

These two conditions are never both true.

*Case 2:* The first factor is negative and second factor is positive.

$$x - 8 \leq 0 \text{ and } x + 4 \geq 0$$

$$x \leq 8 \text{ and } x \geq -4$$

These inequalities both hold for  $-4 \leq x \leq 8$ .

The solution set for  $x^2 \leq 4(x + 8)$  is  $\{x \mid -4 \leq x \leq 8, x \in \mathbb{R}\}$ .

**d)**  $5x^2 \geq 4 - 12x$

$$5x^2 + 12x - 4 \geq 0$$

Use the quadratic formula to find the roots.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

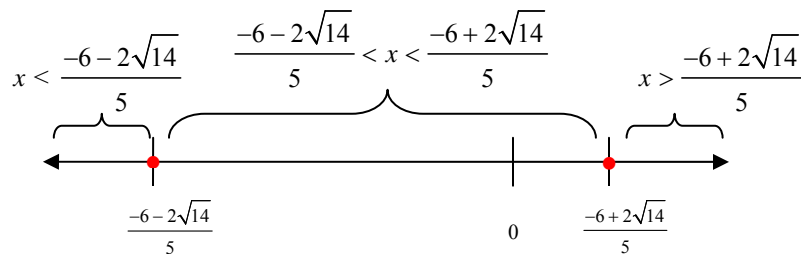
$$x = \frac{-(12) \pm \sqrt{(12)^2 - 4(5)(-4)}}{2(5)}$$

$$x = \frac{-12 \pm \sqrt{224}}{10}$$

$$x = \frac{-6 \pm \sqrt{56}}{5}$$

$$x = \frac{-6 \pm 2\sqrt{14}}{5}$$

The roots are approximately  $-2.70$  and  $0.30$ .



Choose test points in each of the three intervals.

Interval	$x < \frac{-6-2\sqrt{14}}{5}$	$\frac{-6-2\sqrt{14}}{5} < x < \frac{-6+2\sqrt{14}}{5}$	$x > \frac{-6+2\sqrt{14}}{5}$
Test Point	-3	0	1
Substitution	$5x^2 = 5(-3)^2 = 45$ $4 - 12x$ $= 4 - 12(-3)$ $= 40$	$5x^2 = 5(0)^2 = 0$ $4 - 12x$ $= 4 - 12(0)$ $= 4$	$5x^2 = 5(1)^2 = 5$ $4 - 12x$ $= 4 - 12(1)$ $= -8$
Is $5x^2 \geq 4 - 12x$ ?	yes	no	yes

The solution set for  $5x^2 \geq 4 - 12x$  is  $\{x \mid x \leq \frac{-6-2\sqrt{14}}{5} \text{ or } x \geq \frac{-6+2\sqrt{14}}{5}, x \in \mathbb{R}\}$ .

## Chapter 9 Review Page 502 Question 8

a)  $-\frac{3}{4}x^2 + 3x \geq 2$

$$-3x^2 + 12x \geq 8$$

$$3x^2 - 12x + 8 \leq 0$$

Use the quadratic formula to determine the roots.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

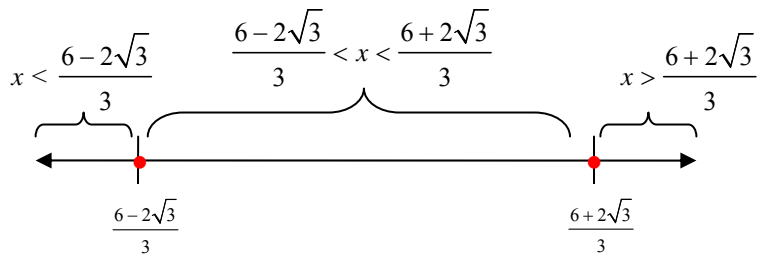
$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(3)(8)}}{2(3)}$$

$$x = \frac{12 \pm \sqrt{48}}{6}$$

$$x = \frac{6 \pm \sqrt{12}}{3}$$

$$x = \frac{6 \pm 2\sqrt{3}}{3}$$

The roots are approximately 0.85 and 3.15.

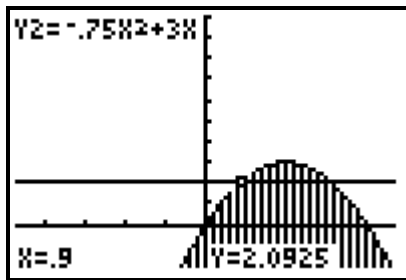


Choose test points in each of the three intervals.

Interval	$x < \frac{6-2\sqrt{3}}{3}$	$\frac{6-2\sqrt{3}}{3} < x < \frac{6+2\sqrt{3}}{3}$	$x > \frac{6+2\sqrt{3}}{3}$
Test Point	0	1	4
Substitution	$-0.75x^2 + 3x$ $= -0.75(0)^2 + 3(0)$ $= 0$	$-0.75x^2 + 3x$ $= -0.75(1)^2 + 3(1)$ $= 2.25$	$-0.75x^2 + 3x$ $= -0.75(4)^2 + 3(4)$ $= 0$
Is $-\frac{3}{4}x^2 + 3x \geq 2$ ?	no	yes	no

The solution set for  $-\frac{3}{4}x^2 + 3x \leq 2$  is  $\{x \mid \frac{6-2\sqrt{3}}{3} \leq x \leq \frac{6+2\sqrt{3}}{3}, x \in \mathbb{R}\}$ .

b)



### Chapter 9 Review Page 502 Question 9

Let  $x$  metres represent the width of the shed. Then its length is  $2x$  metres. For a maximum area of  $18 \text{ m}^2$ :

$$x(2x) \leq 18$$

$$x^2 \leq 9$$

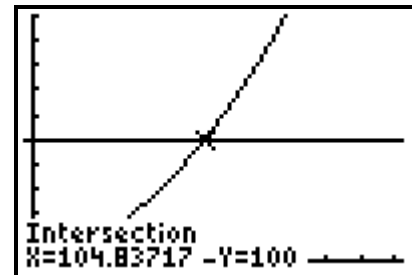
Since  $x$  is a length,  $x \leq 3$ .

The shed has a maximum width of 3 m and maximum length of 6 m.

### Chapter 9 Review Page 502 Question 10

a) Graph the related quadratic function

$y = 0.007v^2 + 0.22v$  and the line  $y = 100$  to determine the value of  $v$  when the distance  $y$  is 100 m.

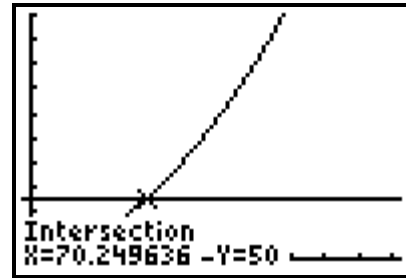


The maximum speed at which David can travel and safely stop his vehicle within 100 m is approximately 104.84 km/h.

b) An inequality for the speeds at which a vehicle can stop in 50 m or less is  $0.007v^2 + 0.22v \leq 50$ .

c) Graph  $y = 0.007v^2 + 0.22v$  and  $y = 50$  and find their point of intersection.

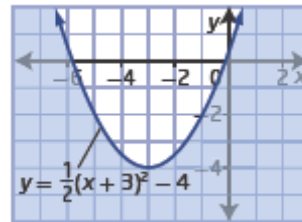
The maximum speed at which David can travel and safely stop his vehicle within 50 m is approximately 70.25 km/h. This answer is not half the speed in part a) because the relationship between speed and stopping distance is quadratic, not linear.



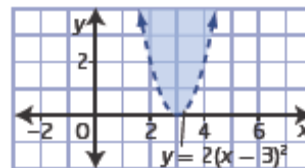
### Chapter 9 Review Page 502 Question 11

a) The boundary is a solid line and the region below the parabola is shaded. So, an inequality for the region is

$$y \leq \frac{1}{2}(x+3)^2 - 4.$$

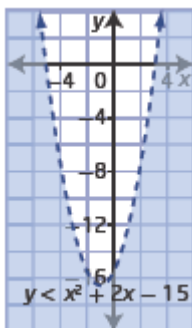


b) The boundary is a broken line and the region above the parabola is shaded. So, an inequality for the region is  $y > 2(x-3)^2$ .

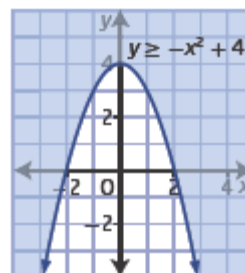


### Chapter 9 Review Page 503 Question 12

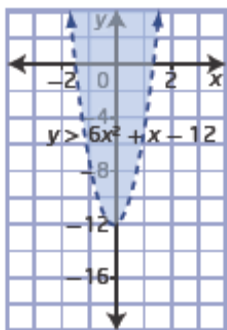
a)  $y < x^2 + 2x - 15$



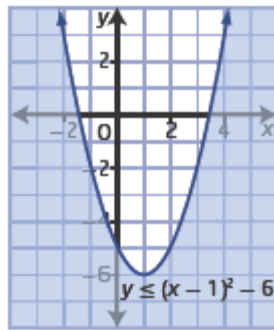
b)  $y \geq -x^2 + 4$



c)  $y > 6x^2 + x - 12$



d)  $y \leq (x - 1)^2 - 6$



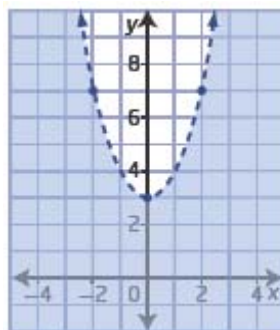
**Chapter 9 Review Page 503 Question 13**

a) Determine an equation for the boundary. The vertex is at  $(0, 3)$ , so the equation has the form  $y = ax^2 + 3$ . Substitute the coordinates of the point  $(2, 7)$  to determine the value of  $a$ .

$$7 = a(2)^2 + 3$$

$$a = 1$$

The inequality for the region shaded is  $y < x^2 + 3$



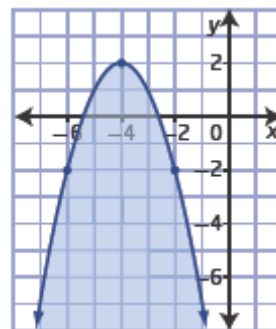
b) Determine an equation for the boundary. The vertex is at  $(-4, 2)$ , so the equation has the form  $y = a(x + 4)^2 + 2$ . Substitute the coordinates of the point  $(-2, -2)$  to determine the value of  $a$ .

$$-2 = a(-2 + 4)^2 + 2$$

$$-4 = 4a$$

$$a = -1$$

The inequality for the region shaded is  $y \leq -(x + 4)^2 + 2$ .



**Chapter 9 Review Page 503 Question 14**

a) The potential wheat production is given by  $y \leq 0.003t^2 - 0.052t + 1.986$ ,  $0 \leq t \leq 20$ ,  $y \geq 0$

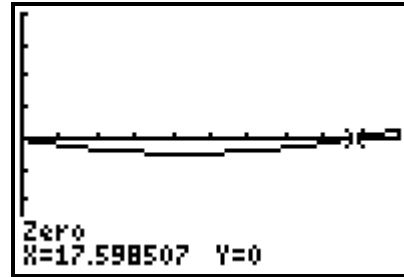


b) For the years in which production is at most 2 t/ha:

$$0.003t^2 - 0.052t + 1.986 \leq 2$$

$$0.003t^2 - 0.052t - 0.014 \leq 0$$

$$3t^2 - 52t - 14 \leq 0$$

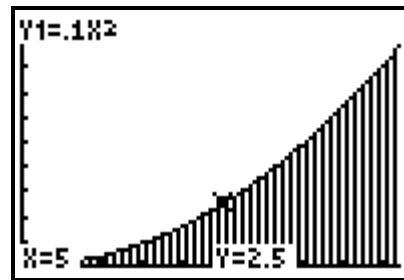


For all years from 1975 until 1992 the production is less than 2 t/ha.

### Chapter 9 Review Page 503 Question 15

a)  $v^2 \geq 10r$

Graph  $r \leq 0.1v^2$  where  $r \geq 0$  and  $v \geq 0$  because the radius and speed must both be non-negative.



b) For  $r = 16$ ,

$$v^2 \geq 10(16)$$

$$v^2 \geq 160$$

$$v \geq \sqrt{160} \text{ or approximately } 12.65$$

Speeds of greater than approximately 12.65 m/s are safe for a loop of radius 16 m.

### Chapter 9 Review Page 503 Question 16

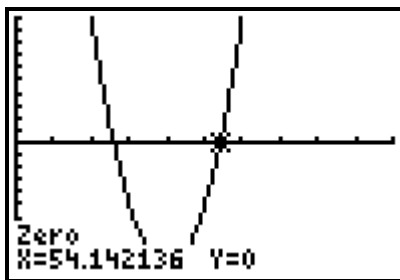
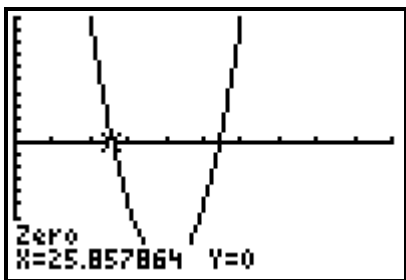
a)  $y = \frac{1}{20}x^2 - 4x + 90$

When the height of the cable,  $y$ , is at least 20 m:

$$\frac{1}{20}x^2 - 4x + 90 \geq 20$$

$$\text{or } \frac{1}{20}x^2 - 4x + 70 \geq 0$$

b) Graph the boundary to determine the roots of  $\frac{1}{20}x^2 - 4x + 70 = 0$ .



The graph is above the  $x$ -axis for  $0 \leq x \leq 25.86$  and  $54.14 \leq x \leq 90$ . The bridge spans from 0 m to 90 m. The solution means that the height cable is at least 20 m above the bridge deck for distances between 0 m and 25.86 m and between 54.14 m and 90 m, measured horizontally from the first support.

## Chapter 9 Practice Test

### Chapter 9 Practice Test Page 504 Question 1

$$3x - 6y < 12$$

$$3x - 12 < 6y$$

$$\frac{1}{2}x - 2 < y$$

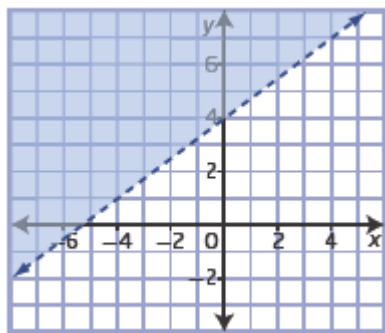
Answer **B** is the equivalent inequality.

### Chapter 9 Practice Test Page 504 Question 2

The  $y$ -intercept of the boundary line is 4 and the slope is  $\frac{3}{4}$ . The equation of the

boundary line is  $y = \frac{3}{4}x + 4$ . So, the inequality for the shaded region is  $y > \frac{3}{4}x + 4$ .

Answer **A** is the correct inequality for the graph.



**Chapter 9 Practice Test      Page 504      Question 3**

$$6x^2 - 7x - 20 < 0$$

$$(3x + 4)(2x - 5) < 0$$

*Case 1:* The first factor is positive and second factor is negative.

$$3x + 4 > 0 \text{ and } 2x - 5 < 0$$

$$x > -\frac{4}{3} \text{ and } x < \frac{5}{2}$$

These two conditions are true for  $-\frac{4}{3} < x < \frac{5}{2}$ .

*Case 2:* The first factor is negative and second factor is positive.

$$3x + 4 < 0 \text{ and } 2x - 5 > 0$$

$$x < -\frac{4}{3} \text{ and } x > \frac{5}{2}$$

These two conditions are never both true.

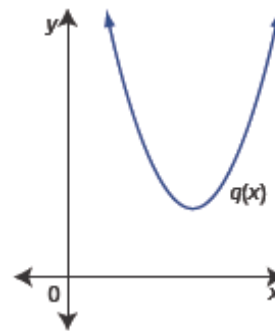
So the solution set is  $\{x \mid -\frac{4}{3} < x < \frac{5}{2}, x \in \mathbb{R}\}$ .

Answer **C** is correct.

**Chapter 9 Practice Test      Page 504      Question 4**

Since the function is entirely above the  $x$ -axis,  $q(x) > 0$  for all real values of  $x$ .

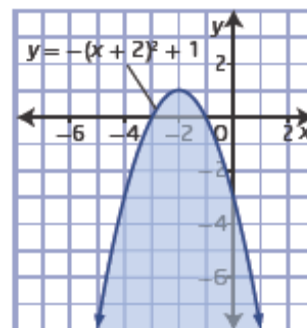
Answer **B** is best.



**Chapter 9 Practice Test      Page 504      Question 5**

Since the boundary is a solid line and the region below it is shaded, the graph shows  $y \leq -(x + 2)^2 + 1$ .

Answer **C** is best.



**Chapter 9 Practice Test      Page 505      Question 6**

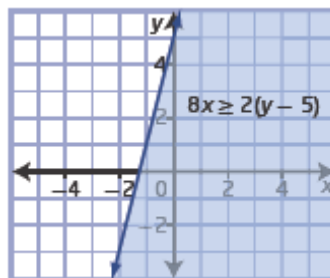
$$8x \geq 2(y - 5)$$

$$4x + 5 \geq y$$

Graph the boundary line  $y = 4x + 5$ .

For the line, the y-intercept is 5 and the slope is 4.

Shade below the line to show  $y \leq 4x + 5$ .



**Chapter 9 Practice Test      Page 505      Question 7**

$$12x^2 < 7x + 10$$

$$12x^2 - 7x - 10 < 0$$

$$(3x + 2)(4x - 5) < 0$$

*Case 1:* The first factor is positive and second factor is negative.

$$3x + 2 > 0 \text{ and } 4x - 5 < 0$$

$$x > -\frac{2}{3} \text{ and } x < \frac{5}{4}$$

These two conditions are true for  $-\frac{2}{3} < x < \frac{5}{4}$ .

*Case 2:* The first factor is negative and second factor is positive.

$$3x + 2 < 0 \text{ and } 4x - 5 > 0$$

$$x < -\frac{2}{3} \text{ and } x > \frac{5}{4}$$

These two conditions are never both true.

So the solution set is  $\{x \mid -\frac{2}{3} < x < \frac{5}{4}, x \in \mathbb{R}\}$ .

**Chapter 9 Practice Test      Page 505      Question 8**

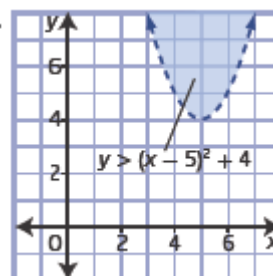
$$y > (x - 5)^2 + 4$$

Consider the boundary parabola,  $y = (x - 5)^2 + 4$ .

The parabola opens upward with vertex at (5, 4).

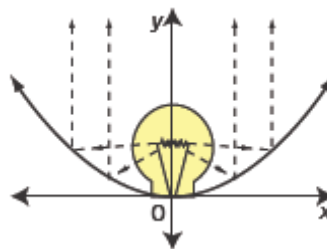
It also passes through (4, 5) and (6, 5).

Use a broken line for the boundary and shade above the line to show  $y > (x - 5)^2 + 4$ .



**Chapter 9 Practice Test      Page 505      Question 9**

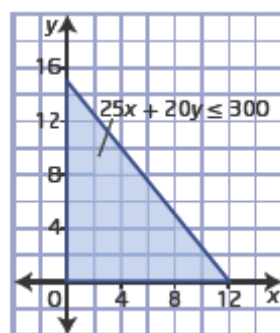
The region above the parabolic reflector is illuminated by the light. The inequality for this region is  $y \geq 0.02x^2$ .



**Chapter 9 Practice Test      Page 505      Question 10**

Let  $x$  represent the number of hours Ben goes scuba diving, at \$25/h, and  $y$  the number of hours that he goes sea kayaking, at \$20/h. He has \$300 to spend so the inequality that represents how he can spend the money is:  
 $25x + 30y \leq 300, x \geq 0, y \geq 0$

The graph shows the region that contains all the possible ways, such as 5 h of scuba diving and 6 h of sea kayaking.



**Chapter 9 Practice Test      Page 505      Question 11**

**a)** Let  $x$  represent the number of pen and ink sketches sold, at \$50 each.

Let  $y$  represent the number of watercolours sold, at \$80 each.

Malik needs an income of at least \$1200 per month. An inequality that models this is  $50x + 80y \geq 1200, x \geq 0, y \geq 0$ .

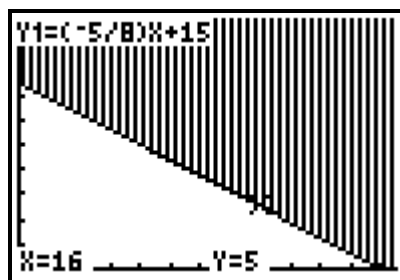
**b)** The boundary line is  $50x + 80y = 1200$  or

$$y = -\frac{5}{8}x + 15.$$

The line has y-intercept 15 and slope  $-\frac{5}{8}$ . Use

a solid line and shade above it, in quadrant I, to show  $50x + 80y \geq 1200, x \geq 0, y \geq 0$ .

Three ordered pairs in the solution region are (0, 15), (10, 10), (16, 5).



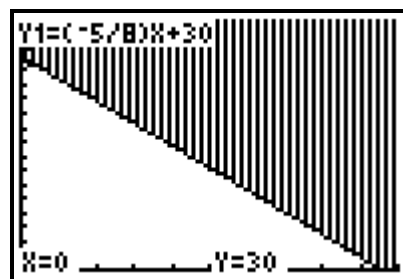
**c)** If Malik needs an income of \$2400, then the inequality will be  $50x + 80y \geq 2400, x \geq 0, y \geq 0$ . The boundary line will be parallel to the one in part b); it will have the same slope but the y-intercept will be 30 instead of 15.

d)  $50x + 80y = 2400$

$$y = -\frac{5}{8}x + 30$$

The line has y-intercept 30 and slope  $-\frac{5}{8}$ .

Use a solid line and shade above it, in quadrant I, to show  $50x + 80y \geq 2400$ ,  $x \geq 0$ ,  $y \geq 0$ .



**Chapter 9 Practice Test      Page 505      Question 12**

Answers may vary. Examples:

a) Work backward: if the solution set is  $\{x \mid -3 \leq x \leq 5, x \in \mathbb{R}\}$  then  $(x + 3)(x - 5) \leq 0$ .  
So,  $f(x) = x^2 - 2x - 15$ .

b) Generalizing the previous answer,  $f(x) = x^2 + mx + nx + mn$ . Any quadratic that opens upward and has  $x$ -intercepts at  $m$  and  $n$ .

c) The form  $f(x) = a(x - p)^2 + q$  is more convenient because this form tells you immediately where the vertex is. So long as  $a$  is positive and the vertex  $(p, q)$  is below the  $x$ -axis then the conditions of part b) are satisfied.

**Chapter 9 Practice Test      Page 505      Question 13**

a) When  $p < 120$ :

$$0.01a^2 + 0.05a + 107 < 120$$

b)  $0.01a^2 + 0.05a + 107 < 120$

$$0.01a^2 + 0.05a - 13 < 0$$

To find the roots, use the quadratic formula to solve  $0.01a^2 + 0.05a - 13 = 0$ .

$$a = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = \frac{-0.05 \pm \sqrt{(0.05)^2 - 4(0.01)(-13)}}{2(0.01)}$$

$$a = \frac{-0.05 \pm \sqrt{0.5225}}{0.02}$$

$$a \approx -38.642 \text{ or } 33.642$$

Since the quadratic opens upward it will be below the  $x$ -axis between the two zeros.

The solution set is  $\{a \mid -38.642 < a < 33.642, a \in \mathbb{R}\}$ .

c) Since  $a$  represents age the negative results do not make sense. A realistic solution set is  $\{a \mid 0 < a \leq 33, a \in \mathbb{R}\}$ .

## Cumulative Review, Chapters 8–9

### Cumulative Review, Chapters 8–9

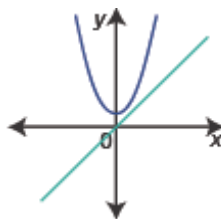
Page 508

### Question 1

a) The graph is a linear-quadratic system, and matches B because the line has  $y$ -intercept 0.

$$y = x^2 + 1$$

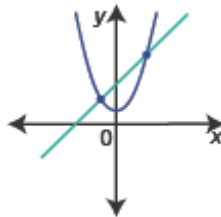
$$y = x$$



b) The graph is a linear-quadratic system, and matches D.

$$y = x^2 + 1$$

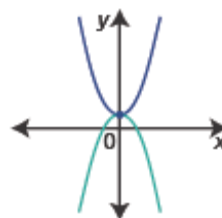
$$y = x + 4$$



c) The graph is a quadratic-quadratic system, and matches A, because both parabolas have  $y$ -intercept 1 and open in different directions.

$$y = x^2 + 1$$

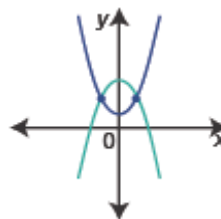
$$y = -x^2 + 1$$



d) The graph is a quadratic-quadratic system, and matches C, because the parabolas have different  $y$ -intercepts and open in different directions.

$$y = x^2 + 1$$

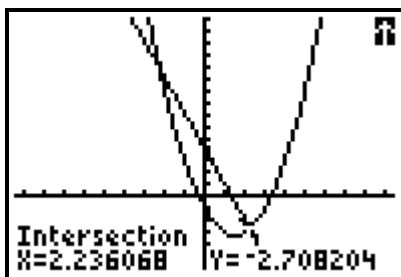
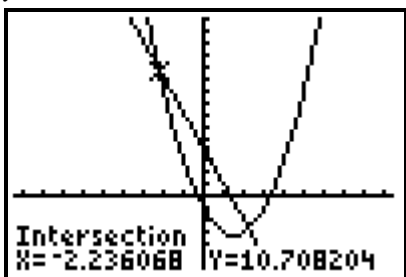
$$y = -x^2 + 4$$



$$3x + y = 4$$

$$y = -3x + 4$$

$$y = x^2 - 3x - 1$$



To the nearest tenth, the solutions are  $(-2.2, 10.7)$  and  $(2.2, -2.7)$ .

a)  $y = -x^2 + 4x + 1$  ①

$$3x - y - 1 = 0 \quad \text{②}$$

Rearrange ②,  $y = 3x - 1$ . Substitute into ①.

$$3x - 1 = -x^2 + 4x + 1$$

$$x^2 - x - 2 = 0$$

$$(x + 1)(x - 2) = 0$$

$$x = -1 \text{ or } x = 2$$

Substitute into ② to find the corresponding y-values.

When  $x = -1$ :

$$3(-1) - y - 1 = 0$$

$$y = -4$$

When  $x = 2$ :

$$3(2) - y - 1 = 0$$

$$y = 5$$

The solutions are  $(-1, -4)$  and  $(2, 5)$ .

b) The ordered pairs are the coordinates of the two points where the line intersects the parabola.

$$y = x^2 + 4 \quad \text{①}$$

$$y = x + b \quad \text{②}$$

Substitute from ② into ①.

$$x + b = x^2 + 4$$

$$x^2 - x + 4 - b = 0$$

Solve using the quadratic formula with  $a = 1$ ,  $b = -1$ , and  $c = 4 - b$ .

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(4-b)}}{2(1)}$$

$$x = \frac{1 \pm \sqrt{4b-15}}{2}$$

**a)** For two solutions the discriminant is greater than 0.

$$4b - 15 > 0$$

$$b > \frac{15}{4} \text{ or } b > 3.75$$

**b)** For exactly one solution the discriminant is 0.

$$4b - 15 = 0$$

$$b = \frac{15}{4} \text{ or } 3.75$$

**c)** For no real solution the discriminant is less than 0.

$$4b - 15 < 0$$

$$b < \frac{15}{4} \text{ or } b < 3.75$$

**Cumulative Review, Chapters 8–9      Page 508      Question 5**

Solving Linear-Quadratic Systems		
Substitution Method	Elimination Method	
Determine which variable to solve for first.	Determine which variable to eliminate.	Multiply the linear equation as needed.
Solve the linear equation for the chosen variable.	Add the new linear equation and the quadratic equation.	
Substitute the expression for the variable into the quadratic equation and simplify.		
Solve the new quadratic equation.		
No Solution	Substitute the value(s) into the original linear equation to determine the corresponding value(s) of the other variable.	

Solving Quadratic-Quadratic Systems		
Substitution Method	Elimination Method	
Solve one quadratic equation for the y-term.	Eliminate the y-term.	Multiply the equations as needed.
Substitute the expression for the y-term into the other quadratic equation and simplify.	Add the new equations.	
Solve the new quadratic equation.		
No Solution	Substitute the value(s) into an original equation to determine the corresponding value(s) of the other variable.	

$$P = 14 + 12t - t^2 \quad \textcircled{1}$$

$$P = 2t + 30 \quad \textcircled{2}$$

Substitute from  $\textcircled{2}$  into  $\textcircled{1}$ .

$$2t + 30 = 14 + 12t - t^2$$

$$t^2 - 10t + 16 = 0$$

$$(t - 2)(t - 8) = 0$$

$$t = 2 \text{ or } t = 8$$

Substitute into  $\textcircled{2}$  to find the corresponding  $P$ -values.

When  $t = 2$ :

$$P = 2(2) + 30$$

$$P = 34$$

When  $t = 8$ :

$$P = 2(8) + 30$$

$$P = 46$$

The two stocks will be the same price after 2 years, when the price of each stock is \$34, and after 8 years when the price of each stock is \$46.

$$y = (x - 4)^2 + 2$$

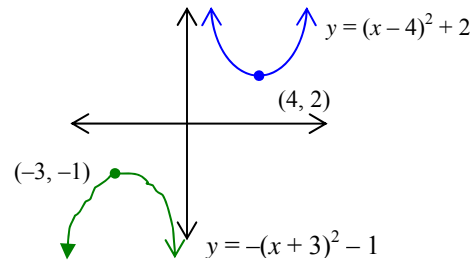
$$y = -(x + 3)^2 - 1$$

From the first equation, the vertex is at (4, 2) and the parabola opens upward.

From the second equation, the vertex is at (-3, -1) and the parabola opens downward.

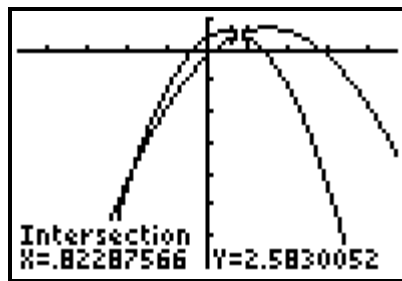
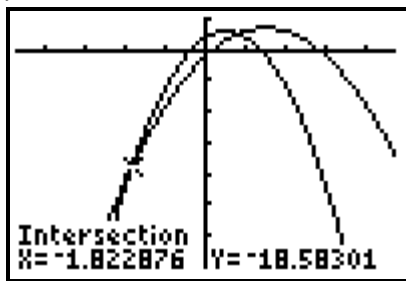
A quick sketch shows that these two parabolas never intersect.

The system has no solution.



$$y = -2x^2 + 6x - 1$$

$$y = -4x^2 + 4x + 2$$



To the nearest tenth the solutions are (-1.8, -18.6) and (0.8, 2.6).

a)  $y = 2x^2 + 9x - 5$  ①

$$y = 2x^2 - 4x + 8$$
 ②

Substitute from ① into ②.

$$2x^2 + 9x - 5 = 2x^2 - 4x + 8$$

$$13x = 13$$

$$x = 1$$

Substitute into ① to determine the corresponding y-value.

$$y = 2(1)^2 + 9(1) - 5$$

$$y = 6$$

The solution is (1, 6).

b)  $y = 12x^2 + 17x - 5$  ①

$$y = -x^2 + 30x - 5$$
 ②

Substitute from ① into ②.

$$12x^2 + 17x - 5 = -x^2 + 30x - 5$$

$$13x^2 - 13x = 0$$

$$13x(x - 1) = 0$$

$$x = 0 \text{ or } x = 1$$

Substitute into ② to determine the corresponding y-values.

When  $x = 0$ :

$$y = -(0)^2 + 30(0) - 5$$

$$y = -5$$

When  $x = 1$ :

$$y = -(1)^2 + 30(1) - 5$$

$$y = 24$$

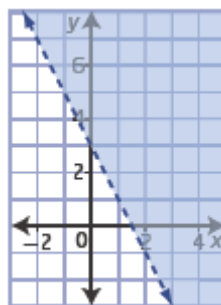
The solutions are  $(0, -5)$  and  $(1, 24)$ .

### Cumulative Review, Chapters 8–9 Page 509 Question 11

**a)** The line has slope  $-2$  and y-intercept  $3$ . So, the equation to the boundary line is  $y = -2x + 3$ . The boundary is a dashed line and the region above it is shaded.

The inequality that describes this graph is  $y > -2x + 3$  or  $2x + y > 3$ .

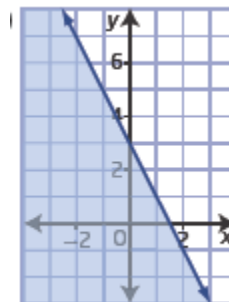
This matches **D**.



**b)** This is the same boundary line as in part a),  $y = -2x + 3$ . The boundary is a solid line and the region below it is shaded.

The inequality that describes this graph is  $y \leq -2x + 3$  or  $2x + y \leq 3$ .

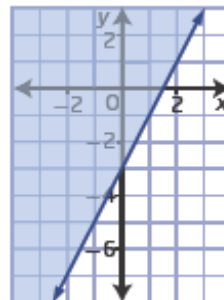
This matches **A**.



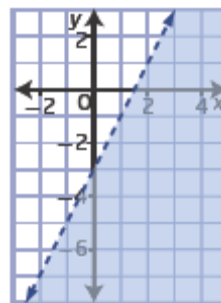
**c)** The line has slope  $2$  and y-intercept  $-3$ . So, the equation to the boundary line is  $y = 2x - 3$ . The boundary is a solid line and the region above it is shaded.

The inequality that describes this graph is  $y \geq 2x - 3$  or  $2x - y \leq 3$ .

This matches **B**.

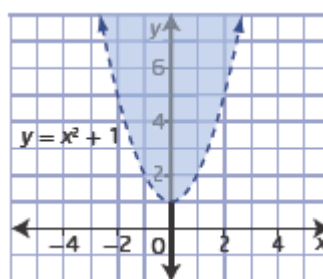


**d)** This is the same boundary line as in part c),  $y = 2x - 3$ . The boundary is a dashed line and the region below it is shaded. The inequality that describes this graph is  $y < 2x - 3$  or  $2x - y > 3$ . This matches **C**.

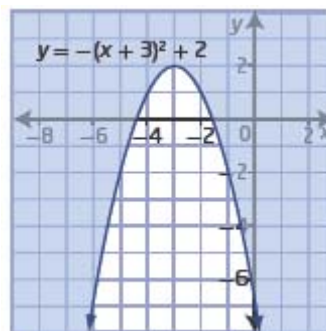


**Cumulative Review, Chapters 8–9      Page 509      Question 12**

**a)** Since the boundary is a dashed line and the region above it is shaded an inequality to describe the graph is  $y > x^2 + 1$ .



**b)** Since the boundary is a solid line and the region above it is shaded an inequality to describe the graph is  $y \geq -(x + 3)^2 + 2$ .



**Cumulative Review, Chapters 8–9      Page 509      Question 13**

**a)** Test  $(0, 0)$  in  $y > x - 2$ .

Left Side = $y$	Right Side = $x - 2$
= $0$	= $0 - 2$
	= $-2$

Left Side  $>$  Right Side

Since this value yields a true statement, shade the region containing  $(0, 0)$ .

**b)** Test  $(2, -5)$  in  $y > x - 2$ .

Left Side = $y$	Right Side = $x - 2$
= $-5$	= $2 - 2$
	= $0$

Left Side  $\not>$  Right Side

Since this value does not check in the inequality, shade the region on the side of the line not containing  $(2, -5)$ .

c) Test  $(-1, 1)$  in  $y > x - 2$ .

$$\begin{aligned}\text{Left Side} &= y \\ &= 1\end{aligned}$$

$$\begin{aligned}\text{Right Side} &= x - 2 \\ &= -1 - 2 \\ &= -3\end{aligned}$$

Left Side  $>$  Right Side

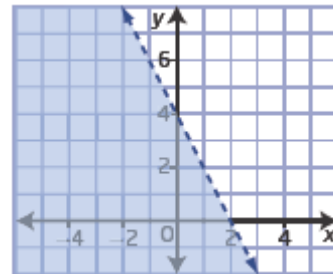
Since this value checks in the inequality, shade the region on the side of the line containing  $(-1, 1)$ .

**Cumulative Review, Chapters 8–9 Page 509**

**Question 14**

The boundary line has slope  $-2$  and  $y$ -intercept  $4$ , so its equation is  $y = -2x + 4$ .

The boundary line is dashed and the region below it is shaded so an inequality for the graph is  $y < -2x + 4$ .



**Cumulative Review, Chapters 8–9 Page 509**

**Question 15**

$$y \geq x^2 - 3x - 4$$

The boundary is the parabola  $y = x^2 - 3x - 4$ .

$$y = (x - 4)(x + 1)$$

The parabola has  $x$ -intercepts  $4$  and  $-1$  and its  $y$ -intercept is  $-4$ .

Use these points to graph the parabola. It opens upward.

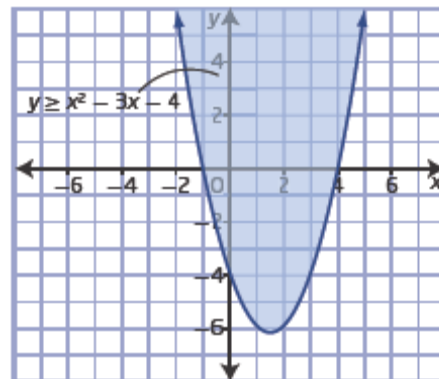
Shade the region above a solid line boundary.

Test  $(0, 0)$ .

Left Side	Right Side
$y$	$x^2 - 3x - 4$
$= 0$	$= (0)^2 - 3(0) - 4$
	$= -4$

Left Side  $>$  Right Side

The correct region is shaded.



$$2x^2 + 9x - 33 \geq 2$$

$$2x^2 + 9x - 35 \geq 0$$

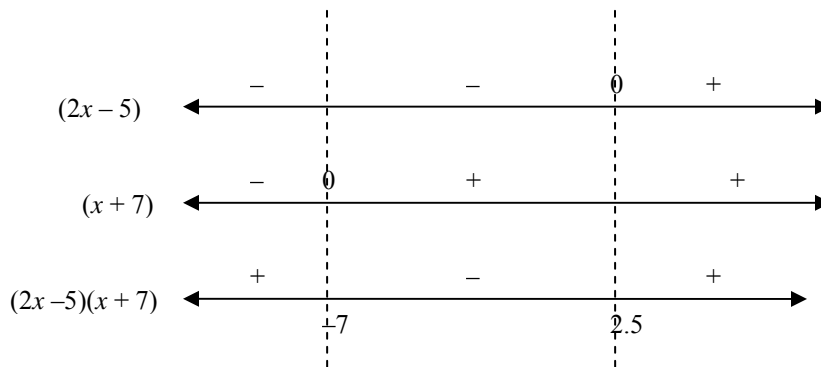
$$(2x - 5)(x + 7) \geq 0$$

Consider key values  $x = 2.5$  and  $x = -7$ , and regions left, right, and between these values.

Substitute 2.5 in  $(x + 7)$ :  $2.5 + 7 = 9.5$  is positive.

Substitute  $-7$  in  $(2x - 5)$ :  $2(-7) - 5 = -19$  is negative.

The signs of the factors in each interval are shown on the diagram below.



The solution set is  $\{x \mid x \leq -7 \text{ or } x \geq 2.5, x \in \mathbb{R}\}$ .

Let  $x$  represent the width of the rectangle. Then,  $500 - x$  represents the length.

For the area to be greater than  $60\,000 \text{ m}^2$ , the following inequality must be true.

$$x(500 - x) > 60\,000$$

$$-x^2 + 500x - 60\,000 > 0$$

$$x^2 - 500x + 60\,000 < 0$$

$$(x - 300)(x - 200) < 0$$

*Case 1:* The first factor is negative and second factor is positive.

$$x - 300 < 0 \text{ and } x - 200 > 0$$

$$x < 300 \text{ and } x > 200$$

These two conditions are true for  $200 < x < 300$ .

*Case 2:* The first factor is positive and second factor is negative.

$$x - 300 > 0 \text{ and } x - 200 < 0$$

$$x > 300 \text{ and } x < 200$$

These two conditions are never both true.

So, the possible widths of the rectangle are between 200 m and 300 m to give an area of greater than  $60\,000 \text{ m}^2$ .

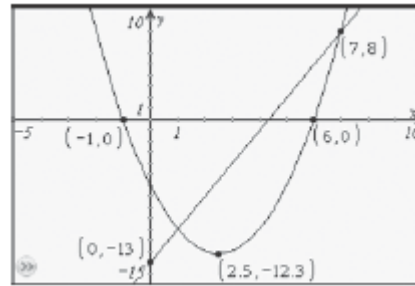
Unit 4 Test

Page 510

Question 1

From the screen image it is clear that  $(7, 8)$  is a point of intersection, so is a solution.

C is the best answer.

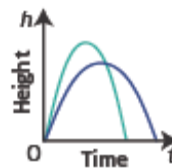


Unit 4 Test

Page 510

Question 2

Graph C is the only graph that shows two racers going over a jump; that is two curves that increase in height, reach a maximum, and then decrease.



Unit 4 Test

Page 510

Question 3

Test ordered pairs  $(1, 3)$  and  $(-3, -5)$  in each equation.

**A**  $y = 3x + 5$

$y = x^2 - 2x - 1$

Test  $(1, 3)$ :

Left Side      Right Side

$$\begin{aligned} y &= 3x + 5 \\ = 3 &= 3(1) + 5 \\ &= 8 \end{aligned}$$

Left Side      Right Side

$$\begin{aligned} y &= x^2 - 2x - 1 \\ = 3 &= (1)^2 - 2(1) - 1 \\ &= -2 \end{aligned}$$

Left Side  $\neq$  Right Side

Left Side  $\neq$  Right Side

Test  $(-3, -5)$ :

Left Side      Right Side

$$\begin{aligned} y &= 3x + 5 \\ = -5 &= 3(-3) + 5 \\ &= -4 \end{aligned}$$

Left Side      Right Side

$$\begin{aligned} y &= x^2 - 2x - 1 \\ = -5 &= (-3)^2 - 2(-3) - 1 \\ &= 14 \end{aligned}$$

Left Side  $\neq$  Right Side

Left Side  $\neq$  Right Side

Therefore,  $(1, 3)$  and  $(-3, -5)$  are not solutions to these equations.

**B**  $y = 2x + 1$

$y = x^2 + 4x - 2$

Test  $(1, 3)$ :

Left Side      Right Side

$$\begin{aligned} y &= 2x + 1 \\ = 3 &= 2(1) + 1 \end{aligned}$$

Left Side      Right Side

$$\begin{aligned} y &= x^2 + 4x - 2 \\ = 3 &= (1)^2 + 4(1) - 2 \end{aligned}$$

$= 3$	$= 3$
Left Side = Right Side	Left Side = Right Side
Test $(-3, -5)$ :	
Left Side	Right Side
$y$	$2x + 1$
$= -5$	$= 2(-3) + 1$
	$= -5$
Left Side = Right Side	Left Side = Right Side

Therefore,  $(1, 3)$  and  $(-3, -5)$  are solutions to these equations.

**C**  $y = x + 2$                        $y = x^2 + 2$

Test  $(1, 3)$ :

Left Side	Right Side	Left Side	Right Side
$y$	$x + 2$	$y$	$x^2 + 2$
$= 3$	$= 1 + 2$	$= 3$	$= (1)^2 + 2$
	$= 3$		$= 3$
Left Side = Right Side	Left Side = Right Side		

Test  $(-3, -5)$ :

Left Side	Right Side	Left Side	Right Side
$y$	$x + 2$	$y$	$x^2 + 2$
$= -5$	$= -3 + 2$	$= -5$	$= (-3)^2 + 2$
	$= -1$		$= 11$
Left Side $\neq$ Right Side	Left Side $\neq$ Right Side		

Therefore,  $(1, 3)$  is a solution but  $(-3, -5)$  is not a solution to these equations.

**D**  $y = 4x - 1$                        $y = x^2 - 3x + 5$

Test  $(1, 3)$ :

Left Side	Right Side	Left Side	Right Side
$y$	$4x - 1$	$y$	$x^2 - 3x + 5$
$= 3$	$= 4(1) - 1$	$= 3$	$= (1)^2 - 3(1) + 5$
	$= 3$		$= 3$
Left Side = Right Side	Left Side = Right Side		

Test  $(-3, -5)$ :

Left Side	Right Side	Left Side	Right Side
$y$	$4x - 1$	$y$	$x^2 - 3x + 5$
$= -5$	$= 4(-3) - 1$	$= -5$	$= (-3)^2 - 3(-3) + 5$
	$= -13$		$= 23$
Left Side $\neq$ Right Side	Left Side $\neq$ Right Side		

Therefore,  $(1, 3)$  is a solution but  $(-3, -5)$  is not a solution to these equations.

The best answer is **B**.

**Unit 4 Test****Page 510****Question 4**

$$y - 5 = 2(x + 1)^2 \rightarrow y = 2(x + 1)^2 + 5 \quad \textcircled{1}$$

$$y - 5 = -2(x + 1)^2 \rightarrow y = -2(x + 1)^2 + 5 \quad \textcircled{2}$$

The graph for  $\textcircled{1}$  has vertex at  $(-1, 5)$  and the parabola opens upward.

The graph for  $\textcircled{2}$  has vertex at  $(-1, 5)$  and the parabola opens downward.

So this pair of equation has one solution.

**B** is the best answer.

**Unit 4 Test****Page 510****Question 5**

$$4x - y \leq 5$$

A point that lies on the boundary line cannot be used to test the solution region.

Test each point, if left side equals right side, then the point is on the line.

**A**  $(-1, 1)$

$$\begin{aligned} \text{Left Side} &= 4x - y & \text{Right Side} &= 5 \\ &= 4(-1) - 1 \\ &= -5 \end{aligned}$$

Left Side  $\neq$  Right Side

This point does not lie on the boundary.

**B**  $(2, 5)$

$$\begin{aligned} \text{Left Side} &= 4x - y & \text{Right Side} &= 5 \\ &= 4(2) - 5 \\ &= 3 \end{aligned}$$

Left Side  $\neq$  Right Side

This point does not lie on the boundary.

**C**  $(3, 1)$

$$\begin{aligned} \text{Left Side} &= 4x - y & \text{Right Side} &= 5 \\ &= 4(3) - 1 \\ &= 11 \end{aligned}$$

Left Side  $\neq$  Right Side

This point does not lie on the boundary.

**D**  $(2, 3)$

$$\begin{aligned} \text{Left Side} &= 4x - y & \text{Right Side} &= 5 \\ &= 4(2) - 3 \\ &= 5 \end{aligned}$$

Left Side = Right Side

This point lies on the boundary, so it cannot be used as a test point for the solution region.

Answer **D** is correct.

## Unit 4 Test

Page 510

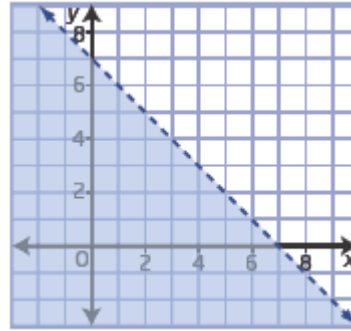
## Question 6

The boundary line has slope  $-1$  and  $y$ -intercept  $7$ , so its equation is

$$y = -x + 7.$$

Since the region below the boundary is shaded the inequality for the graph is  $y < -x + 7$ .

Answer **D** is correct.



## Unit 4 Test

Page 511

## Question 7

Consider  $y \geq 3x^2 + 10x - 8$ . Since “ $y =$ ” is included, the boundary parabola should be a solid line. So, options B and D are not correct. To show  $y \geq$  the region above the boundary is shaded. Test  $(0, 0)$ .

Left Side      Right Side

$$\begin{array}{ll} y & 3x^2 + 10x - 8 \\ = 0 & = 3(0)^2 + 10(0) - 8 \\ & = -8 \end{array}$$

Left Side  $\geq$  Right Side

Answer **A** is correct.

## Unit 4 Test

Page 511

## Question 8

$$y = -\frac{2}{3}x^2 + 2x + 3 \quad \textcircled{1}$$

$$y = x^2 - 4x + 5 \quad \textcircled{2}$$

Substitute from  $\textcircled{1}$  into  $\textcircled{2}$ .

$$-\frac{2}{3}x^2 + 2x + 3 = x^2 - 4x + 5$$

$$-2x^2 + 6x + 9 = 3x^2 - 12x + 15$$

$$5x^2 - 18x + 6 = 0$$

Use the quadratic formula with  $a = 5$ ,  $b = -18$ , and  $c = 6$ .

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-18) \pm \sqrt{(-18)^2 - 4(5)(6)}}{2(5)}$$

$$x = \frac{18 \pm \sqrt{204}}{10}$$

$$x \approx 3.228 \text{ or } x \approx 0.372$$

Substitute in ② to find the corresponding y-values.

When  $x \approx 3.228$ :

$$y = x^2 - 4x + 5$$

$$y = (3.228)^2 - 4(3.228) + 5$$

$$y \approx 2.508$$

When  $x \approx 0.372$ :

$$y = x^2 - 4x + 5$$

$$y = (0.372)^2 - 4(0.372) + 5$$

$$y \approx 3.651$$

To the nearest tenth, the solutions are (3.2, 2.5) and (0.4, 3.7).

Answer **B** is correct.

#### Unit 4 Test

Page 511

#### Question 9

$$-3x^2 + x + 11 < 1$$

$$3x^2 - x - 10 > 0$$

$$(3x + 5)(x - 2) > 0$$

Case 1: Both factors are positive.

$$3x + 5 > 0 \text{ and } x - 2 > 0$$

$$x > -\frac{5}{3} \text{ and } x > 2$$

Both conditions are true for  $x > 2$ .

Case 2: Both factors are negative.

$$3x + 5 < 0 \text{ and } x - 2 < 0$$

$$x < -\frac{5}{3} \text{ and } x < 2$$

Both conditions are true for  $x < -\frac{5}{3}$ .

So, the solution set is  $\{x \mid x < -\frac{5}{3} \text{ or } x > 2, x \in \mathbb{R}\}$ .

Answer **A** is correct.

#### Unit 4 Test

Page 511

#### Question 10

$$y = x^2 - 4x - 2 \quad \text{①}$$

$$y = x - 2 \quad \text{②}$$

If  $(a, 3)$  is a solution, substitute  $y = 3$  into ②.

$$3 = x - 2$$

$$x = 5$$

Check by substituting  $x = 5$  into ①.

$$y = 5^2 - 4(5) - 2$$

$$y = 3$$

The value of  $a$  is 5.

**Unit 4 Test****Page 511****Question 11**

$$y = x^2 - 4x + 6 \quad \textcircled{1}$$

$$y = -x^2 + 6x - 6 \quad \textcircled{2}$$

If  $(a, a)$  is a solution, substitute into  $\textcircled{1}$ .

$$a = a^2 - 4a + 6$$

$$0 = a^2 - 5a + 6$$

$$0 = (a - 3)(a - 2)$$

$$a = 3 \text{ or } a = 2$$

Check  $(3, 3)$ :

In  $\textcircled{1}$

Left Side

$$\begin{aligned} y \\ = 3 \end{aligned}$$

Right Side

$$\begin{aligned} -x^2 + 6x - 6 \\ = -(3)^2 + 6(3) - 6 \\ = 3 \end{aligned}$$

One solution is of the form  $(a, a)$  where the value of  $a$  is **3**.

**Unit 4 Test****Page 511****Question 12**

$$h(t) = -5t^2 + 5t + 4$$

Determine when  $h(t) > 4$ .

$$-5t^2 + 5t + 4 > 4$$

$$-5t^2 + 5t > 0$$

$$5t(t - 1) < 0$$

Since  $t$  represents time, the first factor must be positive and the second negative.

$t > 0$  and  $t < 1$ .

Laurie is above 4 m between 0 s and 1 s, so the length of time is **1 s**.

**Unit 4 Test****Page 512****Question 13**

$$\text{a) } y = 2x \quad \textcircled{1}$$

$$y = \frac{1}{4}x^2 + \frac{3}{2}x \quad \textcircled{2}$$

Substitute from  $\textcircled{1}$  into  $\textcircled{2}$ .

$$2x = \frac{1}{4}x^2 + \frac{3}{2}x$$

$$8x = x^2 + 6x$$

$$0 = x^2 - 2x$$

$$0 = x(x - 2)$$

$$x = 0 \text{ or } x = 2$$

Substitute into ① to determine the corresponding y-values.

$$\text{When } x = 0:$$

$$\text{When } x = 2:$$

$$y = 2(0)$$

$$y = 2(2)$$

$$y = 0$$

$$y = 4$$

The solutions are (0, 0) and (2, 4).

**b)** The point (0, 0) is the starting point. The point (2, 4) is the position of the hole relative to the point from which the golf ball is puttied.

#### Unit 4 Test

#### Page 512

#### Question 14

$$f(x) = x^2 - 6x + 5$$

Since  $g(x)$  is congruent to  $f(x)$  but opens downward, it has the form  $g(x) = -(x - p)^2 + q$ .

The parabola passes through (2, -3) and (7, 12) so substitute these points to form two equations.

$$-3 = -(2 - p)^2 + q \quad \text{①}$$

$$12 = -(7 - p)^2 + q \quad \text{②}$$

From ②,  $q = 12 + (7 - p)^2$ . Substitute in ①.

$$-3 = -(2 - p)^2 + 12 + (7 - p)^2$$

$$-3 = -(4 - 4p + p^2) + 12 + 49 - 14p + p^2$$

$$10p = 60$$

$$p = 6$$

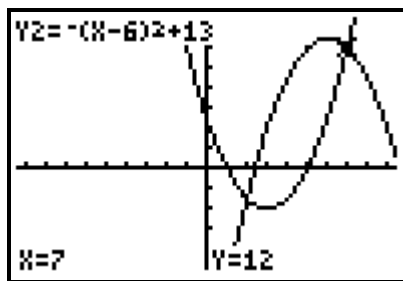
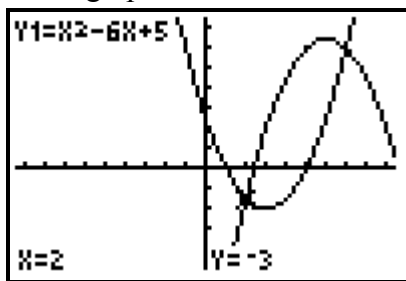
Substitute  $p = 6$  in ① to determine the corresponding value of  $q$ .

$$-3 = -(2 - 6)^2 + q$$

$$q = 13$$

The equation for  $g(x)$ , in vertex form, is  $g(x) = -(x - 6)^2 + 13$ .

Use a graph to check:



#### Unit 4 Test

#### Page 512

#### Question 15

$$4x^2 + 8x + 9 - y = 5 \quad \rightarrow \quad 4x^2 + 8x + 4 = y \quad \text{①}$$

$$3x^2 - x + 1 = y + x + 6 \quad \rightarrow \quad 3x^2 - 2x - 5 = y \quad \text{②}$$

Substitute from ② into ①.

$$4x^2 + 8x + 4 = 3x^2 - 2x - 5$$

$$x^2 + 10x + 9 = 0$$

$$(x + 9)(x + 1) = 0$$

$$x = -9 \text{ or } x = -1$$

Substitute into the first original equation to determine the corresponding y-values.

When  $x = -9$ :

$$4(-9)^2 + 8(-9) + 9 - y = 5$$

$$256 = y$$

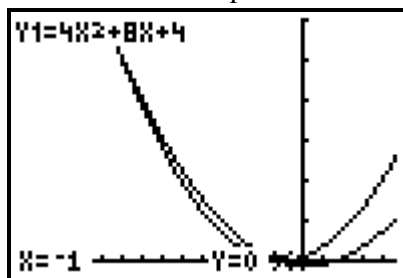
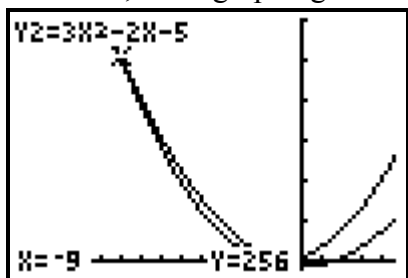
When  $x = -1$ :

$$4(-1)^2 + 8(-1) + 9 - y = 5$$

$$0 = y$$

The solutions are  $(-9, 256)$  and  $(-1, 0)$ .

To check, use a graphing calculator to determine the points of intersection of ① and ②.



#### Unit 4 Test

#### Page 512

#### Question 16

Dolores made an error in the second step, the constant term,  $-8$ , should be  $-12$ . The correct solution is:

$$3x^2 - 5x - 10 > 2$$

$$3x^2 - 5x - 12 > 0$$

$$(3x + 4)(x - 3) > 0$$

$$x = -\frac{4}{3} \text{ or } x = 3$$

Choose test points  $-2$ ,  $0$ , and  $4$  for intervals  $x < -\frac{4}{3}$ ,  $-\frac{4}{3} < x < 3$ , and  $x > 3$ , respectively.

Try  $x = -2$ :

$$\text{Left Side} = 3(-2)^2 - 5(-2) - 10$$

$$= 12$$

$$> 2$$

Values  $x < -\frac{4}{3}$  satisfy the original inequality.

Try  $x = 0$ :

$$\text{Left Side} = 3(0)^2 - 5(0) - 10$$

$$= -10$$

$$< 2$$

Values  $-\frac{4}{3} < x < 3$  do not satisfy the original inequality.

Try  $x = 4$ :

$$\begin{aligned}\text{Left Side} &= 3(4)^2 - 5(4) - 10 \\ &= 18 \\ &> 2\end{aligned}$$

Values  $x > 3$  satisfy the original inequality.

The solution set is  $\{x \mid x < -\frac{4}{3} \text{ or } x > 3, x \in \mathbb{R}\}$ .

**b)** Methods may vary. For example, use cases.

$$3x^2 - 5x - 10 > 2$$

$$3x^2 - 5x - 12 > 0$$

$$(3x + 4)(x - 3) > 0$$

*Case 1:* Both factors are positive.

$$3x + 4 > 0 \text{ and } x - 3 > 0$$

$$x > -\frac{4}{3} \text{ and } x > 3$$

Both conditions are true when  $x > 3$ .

*Case 2:* Both factors are negative.

$$3x + 4 < 0 \text{ and } x - 3 < 0$$

$$x < -\frac{4}{3} \text{ and } x < 3$$

Both conditions are true when  $x < -\frac{4}{3}$ .

The solution set is  $\{x \mid x < -\frac{4}{3} \text{ or } x > 3, x \in \mathbb{R}\}$ .

## Unit 4 Test

Page 512

## Question 17

$$h(t) = -4.9t^2 + 10.4t$$

Determine when  $h(t) > 3$ .

$$-4.9t^2 + 10.4t > 3$$

$$-4.9t^2 + 10.4t - 3 > 0$$

Use the quadratic formula with  $a = -4.9$ ,  $b = 10.4$ , and  $c = -3$ .

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-10.4 \pm \sqrt{(10.4)^2 - 4(-4.9)(-3)}}{2(-4.9)}$$

$$t = \frac{-10.4 \pm \sqrt{49.36}}{-9.8}$$

$$t = \frac{10.4 \pm \sqrt{49.36}}{9.8}$$

$$t \approx 1.778 \text{ or } t \approx 0.344$$

So, the ball is above 3 m for  $1.778 - 0.344$ , or approximately 1.44 s.